AP Calculus AB

Unit #3 Homework—FRQ Practice

Free – Response: 1977 – AB 2

Consider the function f defined by $f(x) = (x^2 - 1)^3$ for all real numbers x.

(a) For what values of x is the function increasing? $(0, +\infty)$

$[(x-1)(x+1)]^3$

$(x-1)^3(x+1)^3$

$f'(x) = 3(x^2-1)^2(2x)$

$= 6x(x^2-1)^2$

(b) Find the x- and y-coordinates of the relative maximum and minimum points. Justify your answer.

rel min $(0,-1)$

no rel max

(c) For what values of is the graph of f concave downward? $(-1, -\sqrt{\frac{1}{5}})$ $(\sqrt{\frac{1}{5}}, 1)$

$f''(x) = 6x \cdot 2(x^2-1)^1(2x) + (x^2-1)^2(6)$

$= 6(x^2-1)[4x^2 + x^2 - 1]$

$= 6(x^2-1)(5x^2-1)$

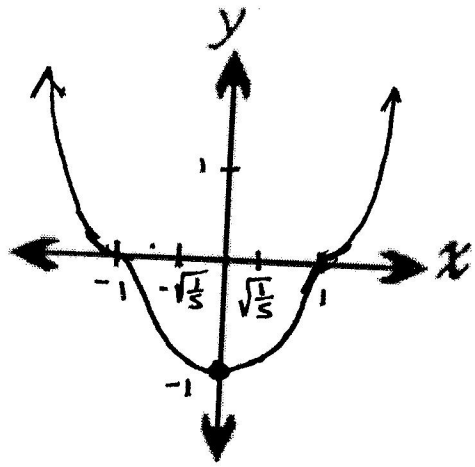
$x^2 - 1 = 0$ $\quad 5x^2 - 1 = 0$

$x = \pm 1$ $\quad x = \pm\sqrt{\frac{1}{5}}$

(d) Using the information found in parts (a), (b), and (c), sketch the graph of f on the axes provided.

f'': $+ \quad - \quad + \quad - \quad +$

$-1 \quad -\sqrt{\frac{1}{5}} \quad \sqrt{\frac{1}{5}} \quad 1$



Free – Response: 1992 – BC4

Let f be the function defined by $f(x) = \begin{cases} 2x - x^2 & \text{for } x \le 1 \\ x^2 + kx + p & \text{for } x > 1 \end{cases}$

a. For what values of k and p will f be continuous and differentiable at $x = 1$?

b. For the values of k and p found in part (a), on what intervals is f increasing?

c. Using the values of k and p found in part (a), find all points of inflection of the graph of f. Justify your answer.

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