1992 BC4
Solution

(a) For continuity at $x = 1$,

$\lim_{x\to 1^-}\left(2x - x^2\right) = f(1) = \lim_{x\to 1^+}\left(x^2 + kx + p\right)$

Therefore $1 = 1 + k + p$

Since f is continuous at $x = 1$ and is piecewise polynomial, left and right derivatives exist.

$f'_-(1) = 0$ and $f'_+(1) = 2 + k$

For differentiability at $x = 1$, $0 = 2 + k$.

Therefore $k = -2$, $p = 2$

(b) $f'(x) = 2 - 2x, \quad x \le 1$

$2 - 2x > 0$

$x < 1$

$f'(x) = 2x - 2, \quad x > 1$

$2x - 2 > 0$

$x > 1$

Since f increases on each of $(-\infty, 1)$ and $(1, \infty)$ and is continuous at $x = 1$, f is increasing on $(-\infty, \infty)$.

(c) $f''(x) = -2, \quad x < 1$

$f''(x) = 2, \quad x > 1$

Since $f''(x) < 0$ on $(-\infty, 1)$ and

$f''(x) > 0$ on $(1, \infty)$ and

$f(1)$ is defined,

$(1, f(1)) = (1, 1)$ is a point of inflection.