

## AP Calculus FRQ's

**Sketch a function that satisfies each of the following conditions:**

$$\lim_{x \rightarrow 0^+} f(x) = 4 \quad f(0) = 2$$

$$\lim_{x \rightarrow 0^-} f(x) = -1 \quad \lim_{x \rightarrow 4^-} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = -1 \quad \lim_{x \rightarrow 4^+} f(x) = -\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

### 1975 AB 1

Given the function  $f$  defined by  $f(x) = \ln(x^2 - 9)$ .

- Describe the symmetry of the graph of  $f$ .
- Find the domain of  $f$ .
- Find all values of  $x$  such that  $f(x) = 0$ .
- Write a formula for  $f^{-1}(x)$ , the inverse function of  $f$ , for  $x > 3$ .

$$i) f(-x) = \ln((-x)^2 - 9) = \ln(x^2 - 9)$$

Since  $f(x) = f(-x) \Rightarrow f$  is even  $\Rightarrow$  s.w.r.t y-axis

$$x^2 - 9 > 0 \text{ pos.}$$

$$(x+3)(x-3) > 0$$

$$\begin{array}{c} + \\ \hline -3 \end{array} \quad \begin{array}{c} - \\ \hline 3 \end{array}$$

$$(-\infty, -3) \cup (3, \infty)$$

$$c) \ln(x^2 - 9) = 0$$

$$e^0 = x^2 - 9$$

$$1 = x^2 - 9$$

$$10 = x^2$$

$$x = \pm \sqrt{10}$$

$$d) x = \ln(y^2 - 9)$$

$$e^x = y^2 - 9$$

$$e^x + 9 = y^2$$

$$y = \pm \sqrt{e^x + 9}$$

$$f^{-1}(x) = \sqrt{e^x + 9}$$

**AP Calculus FRQ's**

**1976 AB 2**

Given the two functions  $f$  and  $h$  such that  $f(x) = x^3 - 3x^2 - 4x + 12$  and  $h(x) = \begin{cases} h(x) = \frac{f(x)}{x-3} & \text{for } x \neq 3 \\ p & \text{for } x = 3 \end{cases}$

- a. Find all zeros of the function  $f$ .

$$\frac{x^3 - 3x^2 - 4x + 12}{x^2(x-3)} = 0$$

$$(x^2-4)(x-3) = 0$$

$$(x-2)(x+2)(x-3) = 0$$

$$x = 2, -2, 3$$

- b. Find the value of  $p$  so that the function  $h$  is continuous at  $x = 3$ . Justify your answer.

$$h(x) = \frac{(x-2)(x+2)(x-3)}{x-3} = (x-2)(x+2) \quad h(3) = (3-2)(3+2) = 5$$

- c. Using the value of  $p$  found in b, determine whether  $h$  is an even function. Justify your answer.

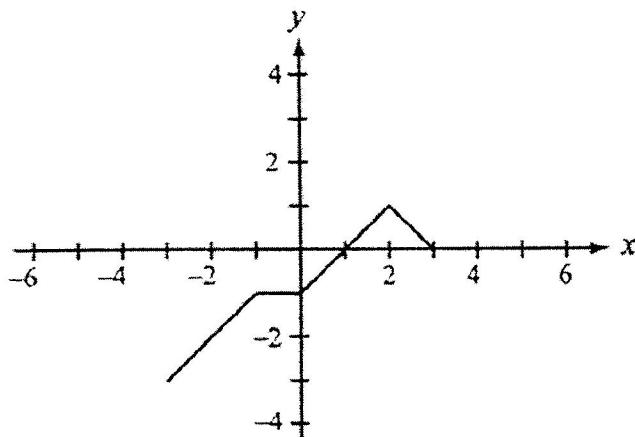
$$h(x) = \begin{cases} x^2 - 4, & x \neq 3 \\ 5, & x = 3 \end{cases}$$

$$h(-x) = (-x)^2 - 4 = x^2 - 4$$

$$h(-x) = h(x) \Rightarrow \text{even}$$

**1970 AB 2**

A function  $f$  is defined on the closed interval from  $-3$  to  $3$  and has the graph shown below.



- a. Sketch the entire graph of  $y = |f(x)|$ .
- b. Sketch the entire graph of  $y = f(|x|)$ .
- c. Sketch the entire graph of  $y = f(-x)$ .
- d. Sketch the entire graph of  $y = f\left(\frac{1}{2}x\right)$ .
- e. Sketch the entire graph of  $y = f(x - 1)$ .

9/7/18

EX1  $f(x) = 2x^2 + 3x$

Find  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ .

$$\lim_{h \rightarrow 0} \frac{2(x+h)^2 + 3(x+h) - (2x^2 + 3x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{2x^2 + 4hx + 2h^2 + 3x + 3h - 2x^2 - 3x}{h}$$

$$\lim_{h \rightarrow 0} (4x + 2h + 3) = 4x + 2(0) + 3 = \boxed{4x + 3}$$

EX2  $f(x) = x^3 - 2$

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - 2 - (x^3 - 2)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - 2 - x^3 + 2}{h}$$

$$\lim_{h \rightarrow 0} (3x^2 + 3hx + h^2) = 3x^2 + 3(0)x + (0)^2 = \boxed{3x^2}$$