

AP Calculus FRQ's

Sketch a function that satisfies each of the following conditions:

$$\lim_{x \rightarrow 0^+} f(x) = 4$$

$$f(0) = 2$$

$$\lim_{x \rightarrow 0^-} f(x) = -1$$

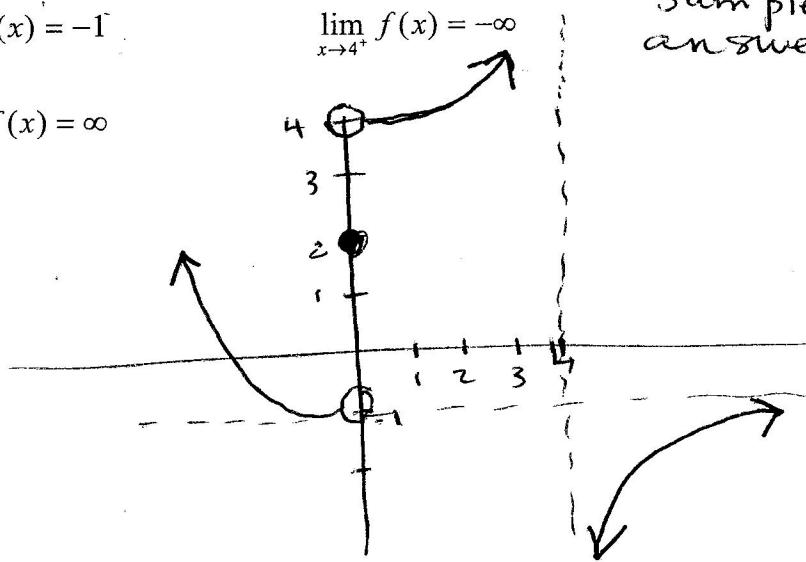
$$\lim_{x \rightarrow 4^-} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = -1$$

$$\lim_{x \rightarrow 4^+} f(x) = -\infty$$

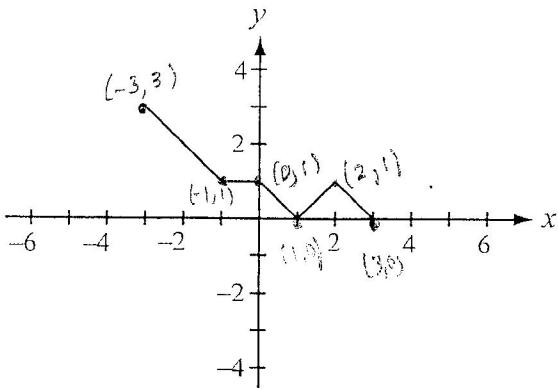
$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

Sample
answer:

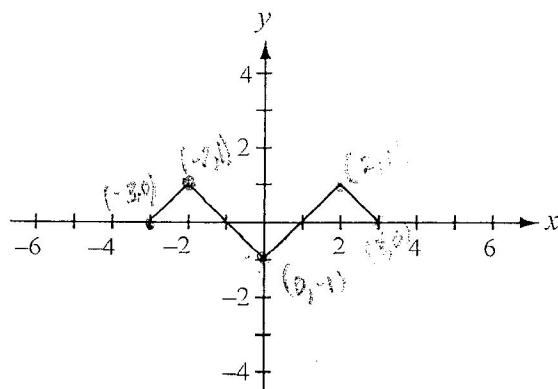


1970 AB2
Solution

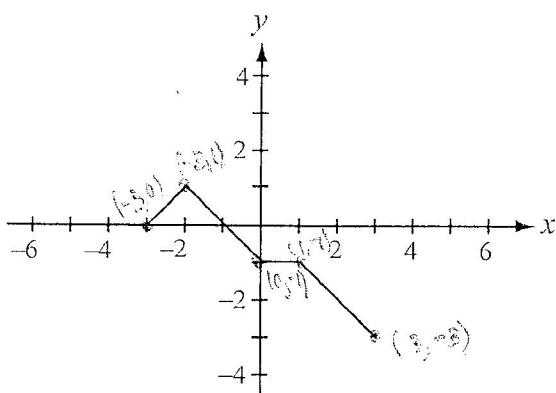
(a) $y = |f(x)|$



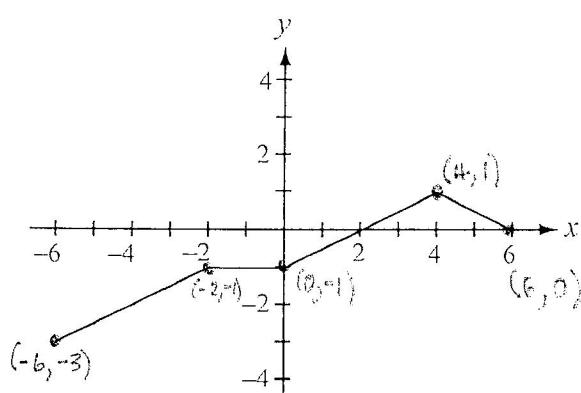
(b) $y = f(|x|)$



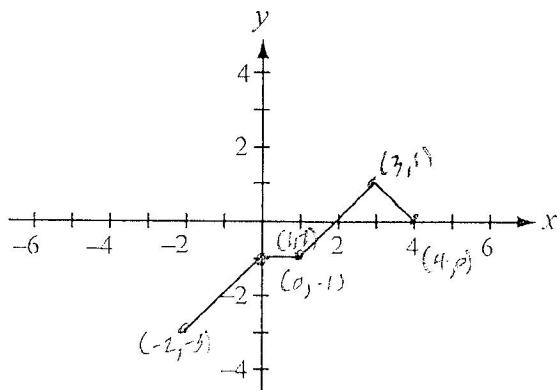
(c) $y = f(-x)$



(d) $y = f\left(\frac{1}{2}x\right)$



(e) $y = f(x-1)$



WHAT DID THE NINJA TURTLES SAY WHEN HANDED THE EXPRESSION $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$?

For each function evaluate $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

1) $f(x) = 3x + 2$ $\lim_{h \rightarrow 0} \frac{[3(x+h) + 2] - (3x + 2)}{h}$ 3	2) $f(x) = 4x - 3$ $\lim_{h \rightarrow 0} \frac{[4(x+h) - 3] - (4x - 3)}{h}$ 4	3) $f(x) = x^2$ $\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$ $2x$
4) $f(x) = x^2 - 5$ $\lim_{h \rightarrow 0} \frac{[(x+h)^2 - 5] - (x^2 - 5)}{h}$ $2x$	5) $f(x) = 3x^2 + x$ $\lim_{h \rightarrow 0} \frac{[3(x+h)^2 + (x+h)] - (3x^2 + x)}{h}$ $6x + 1$	6) $f(x) = x^3$ $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$ $3x^2$
7) $f(x) = 4x^2 + 2x - 7$ $\lim_{h \rightarrow 0} \frac{[4(x+h)^2 + 2(x+h) - 7] - (4x^2 + 2x - 7)}{h}$ $8x + 2$	8) $f(x) = x^4 + 1$ $\lim_{h \rightarrow 0} \frac{[(x+h)^4 + 1] - (x^4 + 1)}{h}$ $4x^3$	

Limits.

A. $f'(x) = 6x + 1$	C. $f'(x) = 4x + 2$	D. $f'(x) = 4x^3$	E. $f'(x) = 2x$	F. $f'(x) = 3x$
I. $f'(x) = 3$	K. $f'(x) = 4x$	R. $f'(x) = 3x^2$	T. $f'(x) = 8x + 2$	V. $f'(x) = 4$

8	3	6	1	2	5	7	1	2	4

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3

Ninja Turtle Puzzle

$$1. \lim_{h \rightarrow 0} \frac{3x + 3h + 2 - 3x - 2}{h} = \lim_{h \rightarrow 0} \frac{3h}{h} = \lim_{h \rightarrow 0} 3 = \boxed{3}$$

$$2. \lim_{h \rightarrow 0} \frac{4x + 4h - 3 - 4x + 3}{h} = \lim_{h \rightarrow 0} \frac{4h}{h} = \lim_{h \rightarrow 0} 4 = \boxed{4}$$

$$3. \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = \boxed{2x}$$

$$4. \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = \boxed{2x}$$

$$5. \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + x + h - 3x^2 - x}{h} = \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + h}{h}$$
$$= \lim_{h \rightarrow 0} (6x + 3h + 1) = \boxed{6x + 1}$$

$$6. \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$
$$= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) = \boxed{3x^2}$$

$$7. \lim_{h \rightarrow 0} \frac{4x^2 + 8xh + 4h^2 + 2x + 2h - 7 - 4x^2 - 2x + 7}{h}$$
$$\lim_{h \rightarrow 0} \frac{8xh + 4h^2 + 2h}{h} = \lim_{h \rightarrow 0} (8x + 4h + 2) = \boxed{8x + 2}$$

$$8. \lim_{h \rightarrow 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 + 1 - x^4 - 1}{h}$$

$$\lim_{h \rightarrow 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} = \lim_{h \rightarrow 0} (4x^3 + 6x^2h + 4xh^2 + h^3)$$
$$= \boxed{4x^3}$$