NOTES--Graph Theory Vocabulary/Euler Paths & Circuits

Important Terms to Know . . .

graph-- finite set of dots & connecting links

vertex-- dot on a graph

edge -- a link on a graph that must connect 2 different vertices

path -- a connected sequence of edges showing a route on a graph that starts at a vertex and ends at a vertex (described by naming in order the vertices visited in transversing it)

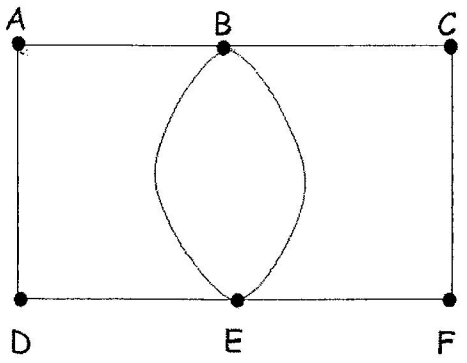
circuit-- a path that starts & ends at the same vertex

valence/degree-- the # of edges that meet at that vertex

connected-- there is a path from one vertex to any other vertex

loop-- counts as 1 edge & contributes 2 to the degree

Example 1 Complete the following information for the graph:



6 vertices 5 edges connected? yes

VERTEX	VALENCE
A	2
B	4
C	2
D	2
E	4
F	2

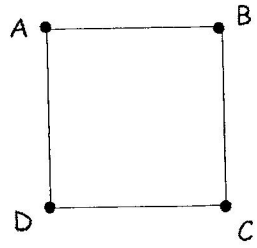
Example 2 Complete the following information for the graph:

2 vertices 2 edges connected? yes

vertex	valence
A	1
B	3

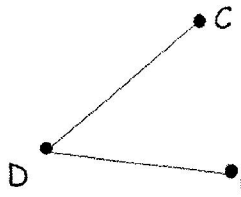
Example 3 Determine if graph is connected or not:

A.



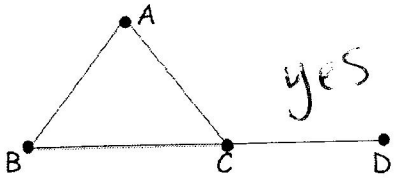
yes

B.



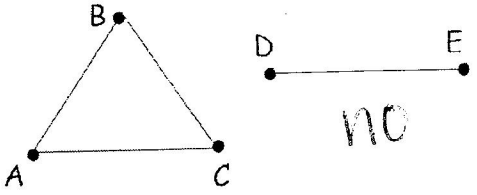
yes

C.



yes

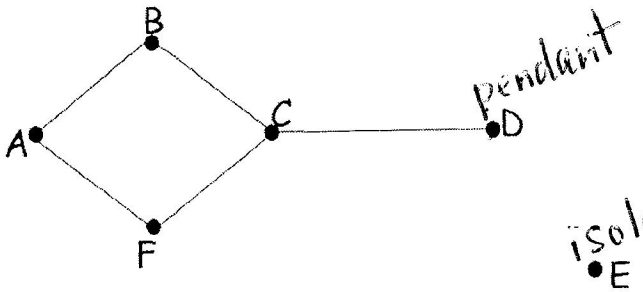
D.

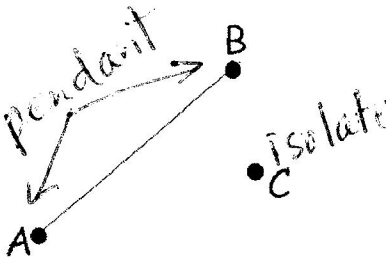


no

pendant vertex-- a vertex with degree 1

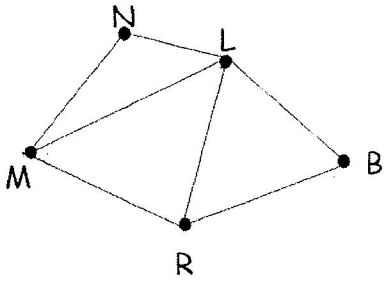
isolated vertex-- a vertex with degree 0





Example 4 Find the requested information about each graph:

A.

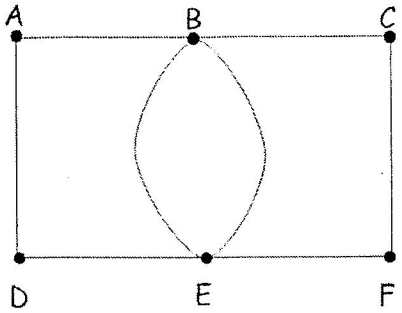


# of vertices = 5

# of edges = 7

some paths from N to B: NLB NMRB
NLRB NLRMN

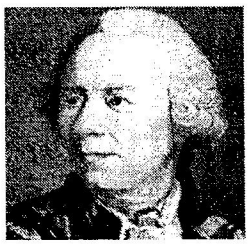
some circuits:
MNLM BLNMRB
MLRM BLRB

B. 

# of vertices = 6

# of edges = 8

some paths from A that cover all of the edges:
ABEBCFEDA
ADEBEFCBA



Euler path-- a path that travels through every edge once

Euler circuit-- a circuit that travels through each edge of a graph only once

Things to know . . .

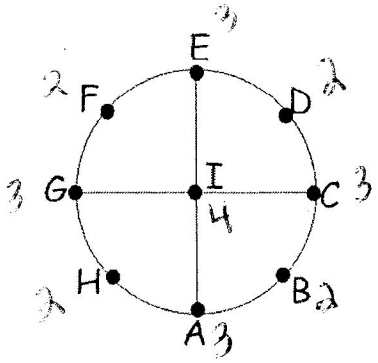
- If a graph has *all even valences*, then it has at least one *Euler circuit*. In other words, if a graph has an odd vertex, then it has no Euler circuit.
- If a graph has *exactly 2 odd vertices*, then it has at least one *Euler path* which starts at one vertex and ends at the other.

Example 5 If possible, name an Euler path and an Euler circuit for each graph:

A. 

Euler path: ABCB
BCBA

Euler circuit: none

B. 

Euler path: none

Euler circuit: none

C. 

Euler path: ABB
BBA

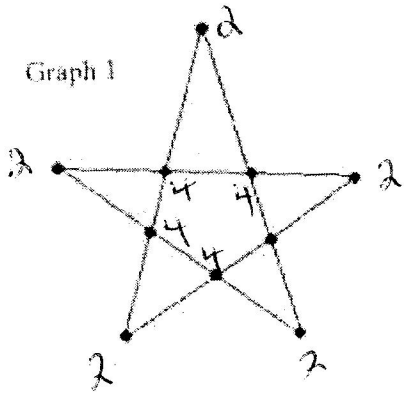
Euler circuit: none

Euler Paths and Circuits

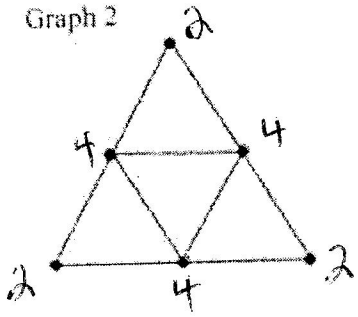
Question: What is the relationship between the nature of the vertices and the kind of path/circuit that the graph contains?

Directions: Complete the chart below using the given graphs. Use the chart to finish the statements.

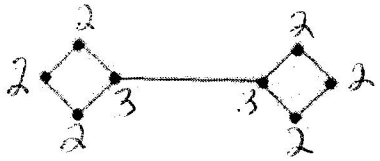
Graph 1



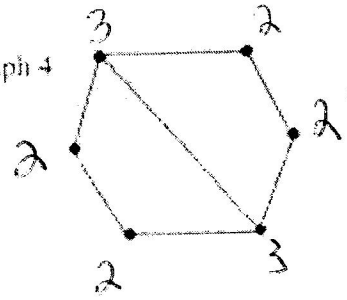
Graph 2



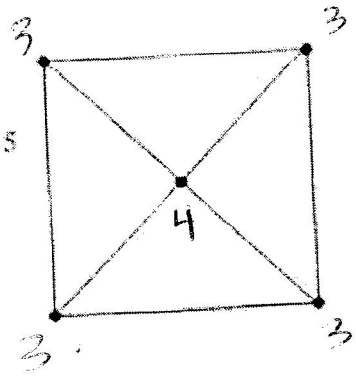
Graph 3



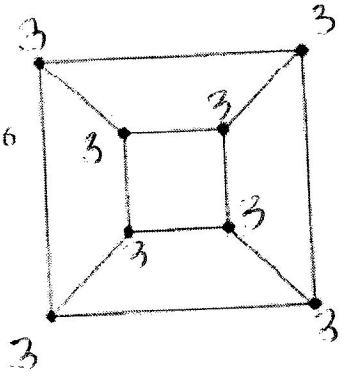
Graph 4



Graph 5



Graph 6



Graph #	Number of vertices with an odd degree	Number of vertices with an even degree	Euler path, Euler circuit, or neither?
1	0	10	EC
2	0	6	EC
3	2	6	EP
4	2	4	EP
5	4	1	neither
6	6	0	neither

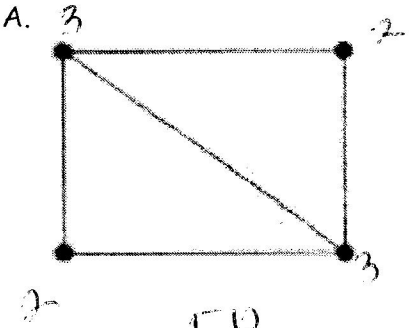
A graph with all even vertices has an Euler circuit.

A graph with 2 odd vertices and some even vertices has an Euler path.

A graph with more than 2 odd vertices has neither.

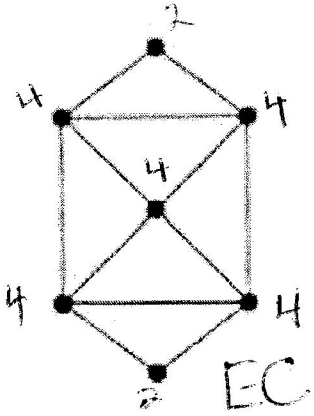
Determine which graphs have Euler paths, Euler circuits, or neither.

A.



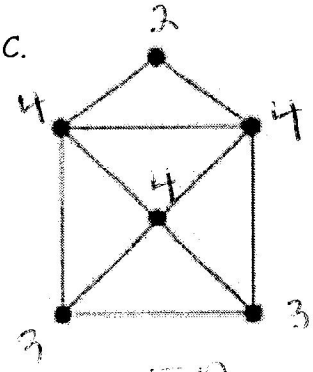
EP

B.



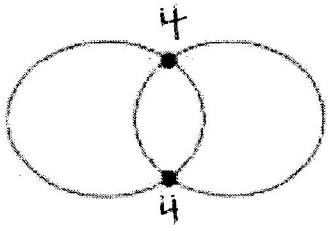
EC

C.



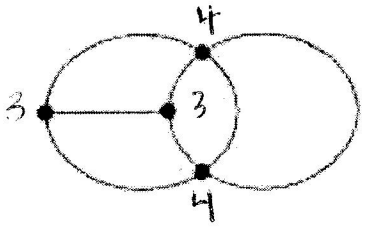
EP

D.



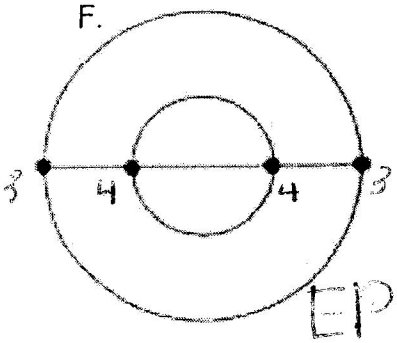
EC

E.



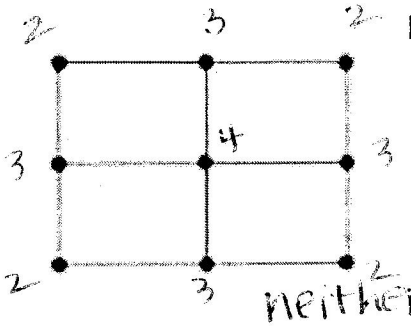
EP

F.



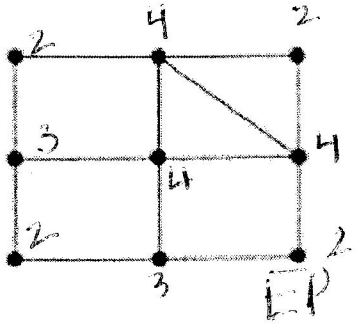
EP

G.



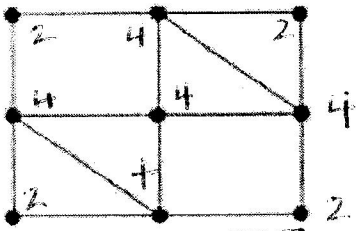
neither

H.



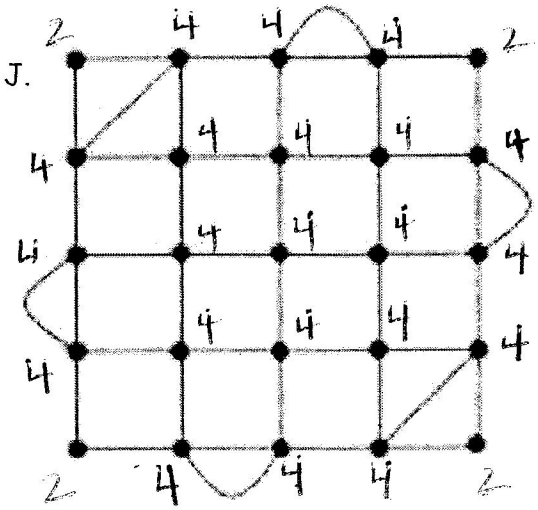
EP

I.



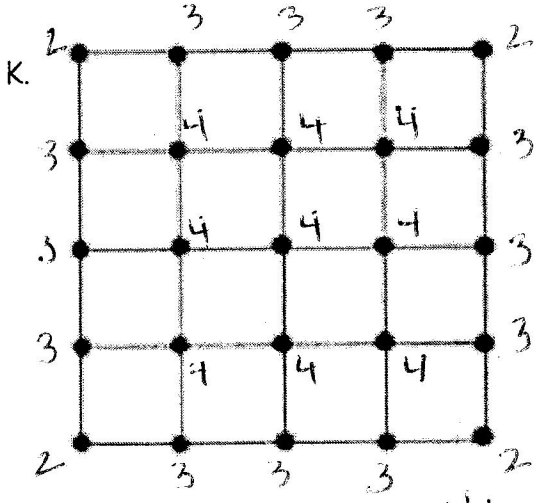
EC

J.



EC

K.



neither