

Graphs of Sinusoids

- The basic graphs of sine and cosine have a period of 2π .
- Changes in amplitude and period as well as phase shifts are nothing more than transformations you've seen before; they have just been given new names for trig functions.
 - > Changes in amplitude are vertical stretches or shrinks/compressions
 - > Changes in period are horizontal stretches or shrinks/compressions
 - > Phase shifts are horizontal (left or right) shifts
 - > These graphs can also be shifted vertically

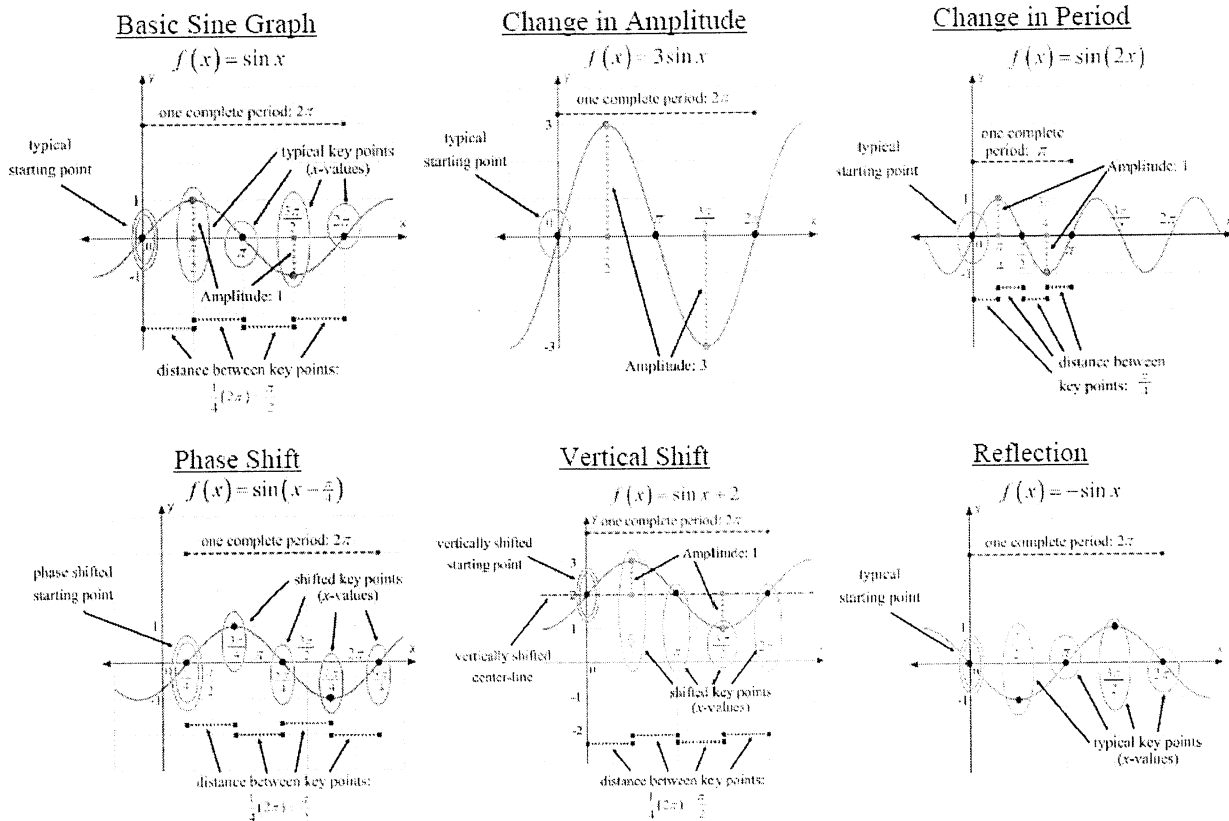
For the graphs of $y = A \sin(Bx - C) + D$ and $y = A \cos(Bx - C) + D$

- Amplitude = $|A|$
- Period = $\frac{2\pi}{B}$
- Phase Shift = $\frac{C}{B}$
 - cos \rightarrow right
 - neg \rightarrow left
- Vertical Shift = D
 - pos \rightarrow up
 - neg \rightarrow down
- Distance Between Key Points* = $\left(\frac{1}{4}\right) \cdot (\text{period})$
- Reflections: Determined by the function used and the values of A and B .
 - For Sine functions – there will be a reflection if the leading coefficient (A) is negative, or if the coefficient of the x -term (B) is negative. If both are negative, there will not be a reflection.
 - For Cosine functions – there will only be a reflection if the leading coefficient (A) is negative. If the coefficient of the x -term (B) is negative, there is some algebraic simplification that must be done, but this will not result in a reflection.

$$y = A \sin B \left(x - \frac{C}{B} \right) + D$$

*Key Points are the points that are at the top or bottom of the graph, or the points on the center-line of the graph

Here are examples of a single change to each of these elements for the basic sine graph.



Example 1

Find the amplitude, period, phase shift, vertical shift, and any reflection.

$$y = -2 \cos 4 \left(x + \frac{\pi}{4} \right) \quad \text{amp} = |-2| = 2 \quad \text{p.s. } -\frac{\pi}{4} \text{ left } \frac{\pi}{4}$$

$$y = -2 \cos(4x + \pi) \quad \text{per} = \frac{2\pi}{4} = \frac{\pi}{2} \quad \text{refl. over x-axis}$$

$$-2 \cos(4x - \pi) \quad \text{v.s. none}$$

Example 2

Find the amplitude, period, phase shift, vertical shift, and any reflection. Then graph one complete period.

A. $y = 3 \sin(6\pi x)$

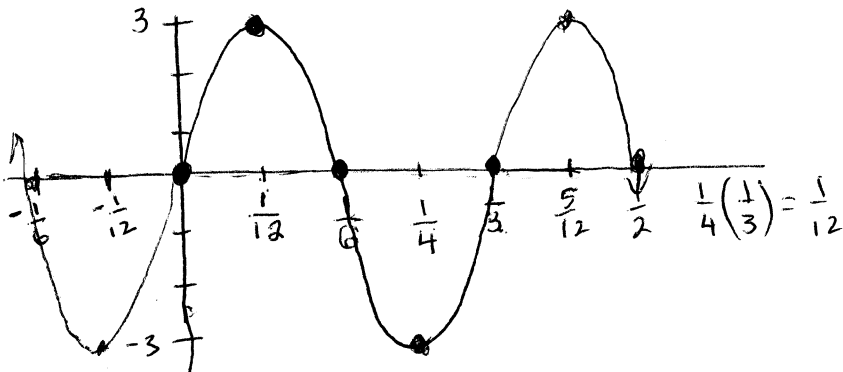
amp = $|3| = 3$

per = $\frac{2\pi}{6\pi} = \frac{1}{3}$

p.s. none

v.s. none

refl. none



B. $y = -2 \cos(x) + 3$

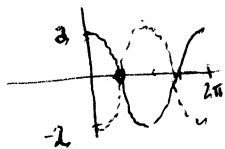
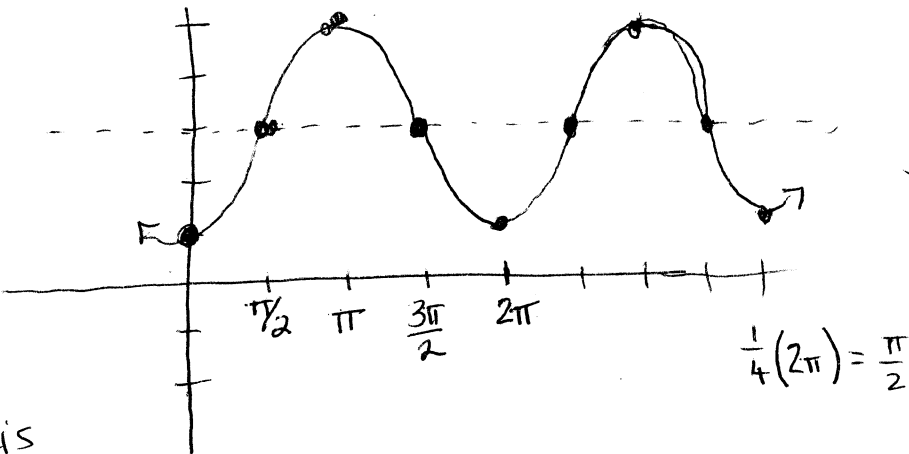
amp = $|-2| = 2$

per = $\frac{2\pi}{1} = 2\pi$

p.s. none

v.s. up 3

refl. over x-axis



C. $y = -3 \sin\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 2$ $c = -\frac{\pi}{4}$

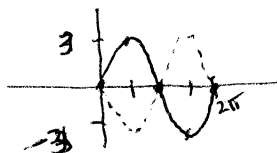
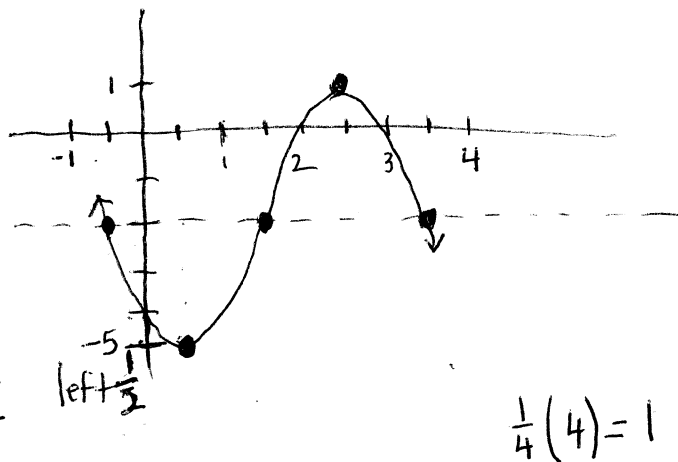
amp = $|-3| = 3$ $b = \frac{\pi}{2}$

per = $\frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4$

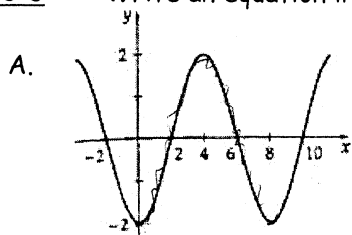
p.s. = $\frac{-\frac{\pi}{4}}{\frac{\pi}{2}} = -\frac{\pi}{4} \cdot \frac{2}{\pi} = -\frac{1}{2}$ left $\frac{1}{2}$

v.s. = down 2

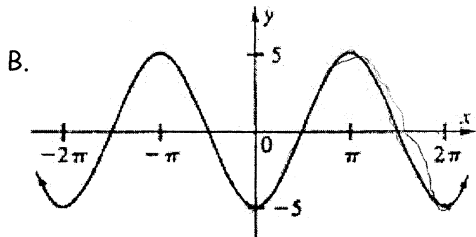
refl. over x-axis



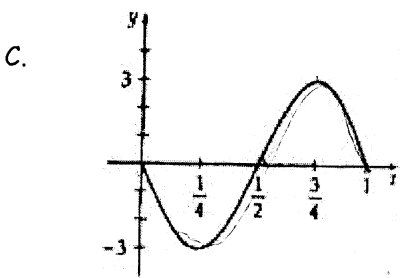
Example 3 Write an equation in the form $y = A\sin(Bx)$ or $y = A\cos(Bx)$ for each graph.



cosine
 amp = $\frac{1}{2}(4) = 2$ refl. over x-axis "A" = -2
 per = 8 $\frac{2\pi}{B} = 8$ $8B = 2\pi$ $B = \frac{2\pi}{8} = \frac{\pi}{4}$
 $y = -2\cos\left(\frac{\pi}{4}x\right)$



cosine
 amp = $\frac{1}{2}(10) = 5$ refl over x-axis A = -5
 per = 2π $\frac{2\pi}{B} = 2\pi$ B = 1
 $y = -5\cos(x)$



sine
 amp = $\frac{1}{2}(6) = 3$ refl. over x-axis A = -3
 per = 1 $\frac{2\pi}{B} = 1$
 $B = 2\pi$
 $y = -3\sin(2\pi x)$

Example 4 Write an equation for each description or graph.

A. sine function; amplitude = 2, period = $\frac{\pi}{3}$, shifted down 1 unit

A = 2 $\frac{2\pi}{B} = \frac{\pi}{3}$ D = -1
 $\pi B = 6\pi$
 B = 6

$$y = 2\sin(6x) - 1$$

B. cosine function; amplitude = 4, period = 3, shifted right $\frac{\pi}{8}$ units, shifted up 2 units

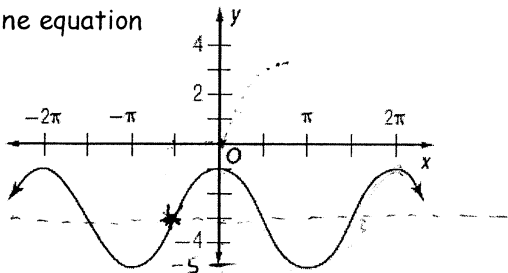
A = 4 $\frac{2\pi}{B} = 3$
 $2\pi = 3B$
 $B = \frac{2\pi}{3}$

p.s. = $\frac{\pi}{8}$

D = 2

$$y = 4\cos\left(\frac{2\pi}{3}\left(x - \frac{\pi}{8}\right)\right) + 2$$

C. sine equation

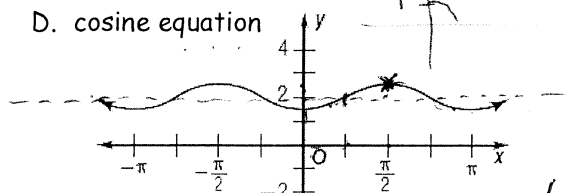


amp = $\frac{1}{2}(4) = 2$ $y = 2\sin\left(x + \frac{\pi}{2}\right) - 3$

per = 2π $\frac{2\pi}{B} = 2\pi$ B = 1

left $\frac{\pi}{2}$ down 3

D. cosine equation



$$y = \frac{1}{2}\cos 2\left(x - \frac{\pi}{2}\right) + 2$$

amp = $\frac{1}{2}(1) = \frac{1}{2}$

per = π $\frac{2\pi}{B} = \pi$ $\pi B = 2\pi$
 B = 2

right $\frac{\pi}{2}$ up 2