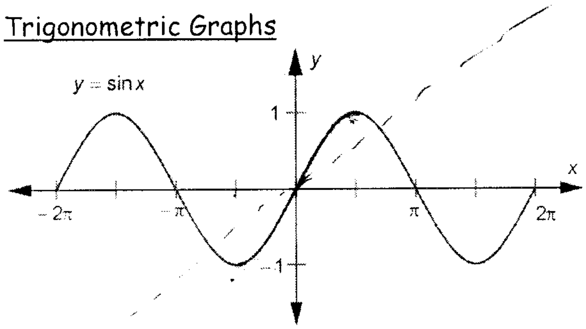
NOTES--Inverse Trig Functions

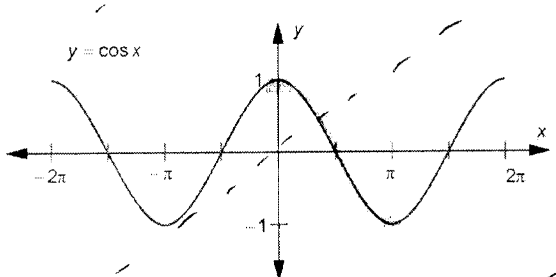
Review of Functions and Their Inverse Functions

1) A function must be one-to-one (any horizontal line intersects it at most once) in order to have an inverse function.
2) The graph of an inverse function is the reflection of the original function about the line y = x.
3) If (x, y) is a point on the graph of the original function, then (y, x) is a point on the graph of the inverse function.
4) The domain and range of a function and its inverse are interchanged.

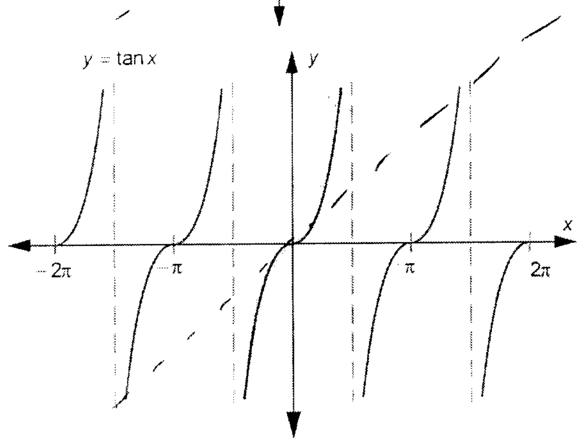
Trigonometric Graphs



D: $[-\pi/2, \pi/2]$
R: $[-1, 1]$

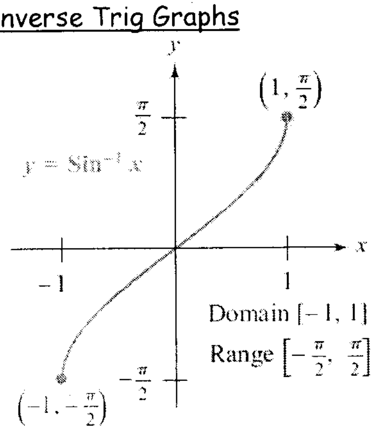


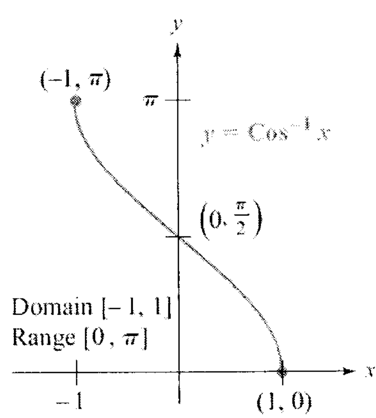
D: $[0, \pi]$
R: $[-1, 1]$

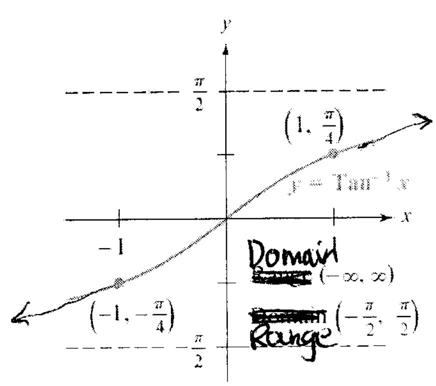


D: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
R: $(-\infty, \infty)$

Inverse Trig Graphs







Trig function	Restricted domain	Inverse trig function	Principle value range	
$y = \sin x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$y = \arcsin x$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	* 4th, 1st
$y = \cos x$	$0 \le x \le \pi$	$y = \arccos x$	$0 \le y \le \pi$	1st, 2nd
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$y = \arctan x$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	* 4th, 1st

Example 1 Evaluate without a calculator.

A. $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

B. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

C. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

D. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

E. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

F. $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

G. $\tan^{-1}(1) = \frac{\pi}{4}$

H. $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

I. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

Example 2 Use a calculator to find:

a. $\sin^{-1}(0.75) = .85$

b. $\tan^{-1}(0.3) = .29$

c. $tan^{-1}(2) = 1.11$

d. $\sin^{-1}(2) =$ undefined
domain is $[-1, 1]$

Example 3 Find:

a. $\sin(\tan^{-1}(1)) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

b. $\cos^{-1}\left(\cos\frac{7\pi}{4}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

c. $arccos\left(\tan\frac{\pi}{4}\right) = \arccos(1) = 0$