Implicit Differentiation

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→ eqn is not solved for y $\quad 12x^2+y^2=144$

Steps: 1) Differentiate both sides w.r.t x

algebra
2) Put terms with $\frac{dy}{dx}$ on one side, everything else on other side
3) Factor out $\frac{dy}{dx}$
4) Solve for $\frac{dy}{dx}$

Ex 1 Find each derivative.

A. $12x^2+y^2=144$

$$24x+2y\cdot\frac{dy}{dx}=0$$

$$2y\frac{dy}{dx}=-24x$$

$$\frac{dy}{dx}=\frac{-24x}{2y}=\frac{-12x}{y}$$

B. $x^2y+y^2x=-2$

$$x^2\cdot 1\frac{dy}{dx}+y\cdot 2x+y^2\cdot 1+x\cdot 2y\frac{dy}{dx}=0$$

$$x^2\frac{dy}{dx}+2xy\frac{dy}{dx}=-y^2-2xy$$

$$\frac{dy}{dx}(x^2+2xy)=-y^2-2xy$$

$$\frac{dy}{dx}=\frac{-y^2-2xy}{x^2+2xy}$$

C. $2x^3y^3 - y = x^{2/3}$

$$2x^3 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot 6x^2 - 1\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}}$$

$$6x^3y^2\frac{dy}{dx} - 1\frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}} - 6x^2y^3$$

$$\frac{dy}{dx}\left(6x^3y^2 - 1\right) = \frac{2}{3\sqrt[3]{x}} - 6x^2y^3$$

$$\frac{dy}{dx} = \frac{\frac{2}{3\sqrt[3]{x}} - 6x^2y^3}{6x^3y^2 - 1}$$

D. $y = \sin(xy)$

$$1\frac{dy}{dx} = \cos(xy) \cdot \left[x \cdot 1\frac{dy}{dx} + y \cdot 1\right]$$

$$1\frac{dy}{dx} = x \cdot \cos(xy) \cdot \frac{dy}{dx} + y \cdot \cos(xy)$$

$$1\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$$

$$\frac{dy}{dx}\left(1 - x\cos(xy)\right) = y\cos(xy)$$

$$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

EX2 $3x + xy^3 = 4$

Find the slope of the curve at the point $(1,1)$.

deriv.

deriv: $3 + x \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 1 = 0$

$$3xy^2 \frac{dy}{dx} = -y^3 - 3$$

$$\frac{dy}{dx} = \frac{-y^3 - 3}{3xy^2}$$ general slope

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-(1)^3 - 3}{3(1)(1)^2} = \frac{-4}{3}$$