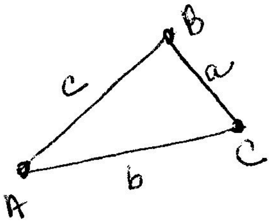
Notes (Section 5.5)-Law of Sines

The Law of Sines (To be used when given 2 angles and a side or 2 consecutive sides and then an angle) (ASA, AAS, or SSA)



In any ΔABC with angles $A, B,$ and C opposite sides $a, b,$ and, c respectively, the following equation is true:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex 1: Solve ΔABC: A = 50°, B = 62°, a = 4.

AAS

$\frac{\sin 50^\circ}{4} = \frac{\sin 62^\circ}{b} = \frac{\sin 68^\circ}{c}$

$b \sin 50^\circ = 4 \sin 62^\circ$ $\quad c \cdot \sin 50^\circ = 4 \sin 68^\circ$

$b = \frac{4 \sin 62^\circ}{\sin 50^\circ} = 4.61$ $\quad c = \frac{4 \sin 68^\circ}{\sin 50^\circ} = 4.84$

$C = 68^\circ, b = 4.61, c = 4.84$

Ex 2: In triangle ΔPQR, m∠P = 36°, m∠Q=48°, and p=8.

AAS

$R = 96^\circ, q = 10.11, r = 13.54$

$\frac{\sin 36^\circ}{8} = \frac{\sin 48^\circ}{q} = \frac{\sin 96^\circ}{r}$

$q \cdot \sin 36^\circ = 8 \sin 48^\circ$

$q = \frac{8 \sin 48^\circ}{\sin 36^\circ} = 10.11$

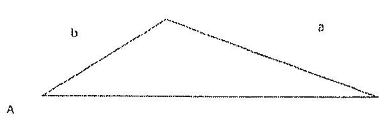
$r \sin 36^\circ = 8 \sin 96^\circ$ $\quad r = \frac{8 \sin 96^\circ}{\sin 36^\circ} = 13.5$

The Ambiguous Case: When given the measures of 2 sides and the angle opposite one of those sides (SSA), it is possible that two, one or no triangle is formed by this angle and sides.

Here is a summary of possibilities:

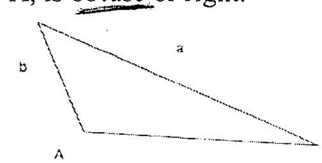
1. If the angle, A, (given angle) is acute (less than 90°) and the side opposite the angle, a, is greater than or equal to the other side, b: $a > b$

There can only be one solution

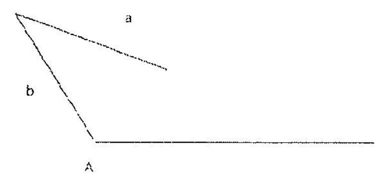


2. If the angle, A, is obtuse or right:

There is one solution if $a > b$



There is no solution if $a \le b$



Ex 3: $\angle C = 97^\circ$, $c = 45$, $a = 39$

case 2 obtuse 45 > 39 one soln SSA

$\frac{\sin 97^\circ}{45} = \frac{\sin A}{39}$

$A = 59.3^\circ$, $B = 23.7^\circ$, $b = 18.19$

$45 \cdot \sin A = 39 \sin 97^\circ$

$A = \sin^{-1}\left(\frac{39 \sin 97^\circ}{45}\right) = 59.3^\circ$

$B = 180^\circ - 97^\circ - 59.3^\circ = 23.7^\circ$

$\frac{\sin 23.7^\circ}{b} = \frac{\sin 97^\circ}{45}$ $\quad b \cdot \sin 97^\circ = 45 \sin 23.7^\circ$

$b = \frac{45 \sin 23.7^\circ}{\sin 97^\circ} = 18.19$

Ex 4: $\angle A = 47^\circ$, $a = 20$, $b = 12$

SSA acute 20 > 12 case 1

$\frac{\sin 47^\circ}{20} = \frac{\sin B}{12}$

$20 \sin B = 12 \sin 47^\circ$

$B = \sin^{-1}\left(\frac{12 \sin 47^\circ}{20}\right) = 26.0^\circ$

$C = 107.0^\circ$

$\frac{\sin 107.0^\circ}{c} = \frac{\sin 47^\circ}{20}$

$c \cdot \sin 47^\circ = 20 \sin 107.0^\circ$

$c = \frac{20 \sin 107.0^\circ}{\sin 47^\circ} = 26.16$

$B = 26.0^\circ$, $C = 107.0^\circ$, $c = 26.16$

Ex 5: $\angle A = 114^\circ$, $a = 21$, $b = 32$

case 2 SSA obtuse 21 < 32

$\frac{\sin 114^\circ}{21} = \frac{\sin B}{32}$

$B = \sin^{-1}\left(\frac{32 \sin 114^\circ}{21}\right)$

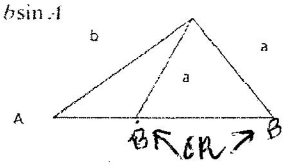
no soln

$B = 141.4^\circ$ $C = 2.6^\circ$ $c = 1.26$
$B = 38.6^\circ$ $C = 105.4^\circ$ $c = 26.25$

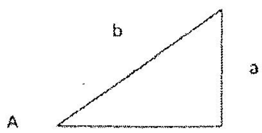
If the angle, A, is acute (less than 90°), the height is bsinA
and the side opposite the angle, a, is less than the other side, b:

$a < b$

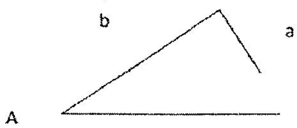
There are two solutions if $a > b\sin A$



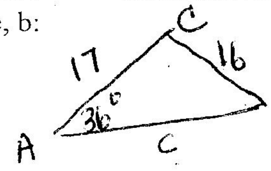
There is one solution if $a = b\sin A$



There is no solution if $a < b\sin A$



Ex6: Solve ΔABC: ~~$\angle A = 36^\circ$, $b = 17$, $a = 16$~~



SSA
16 < 17 case 3
$17\sin 36^\circ = 9.99$
$16 > 9.99$ 2 solns

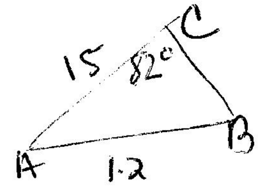
$$\frac{\sin 36^\circ}{16} = \frac{\sin B}{17}$$

$$B = \sin^{-1}\left(\frac{17\sin 36^\circ}{16}\right) = 38.6^\circ \text{ OR } 180^\circ - 38.6^\circ = 141.4^\circ$$

$$C = 180^\circ - 36^\circ - 38.6^\circ = 105.4^\circ$$

$$\frac{\sin 105.4^\circ}{c} = \frac{\sin 36^\circ}{16} \qquad c = \frac{16\sin 105.4^\circ}{\sin 36^\circ} = 26.25$$

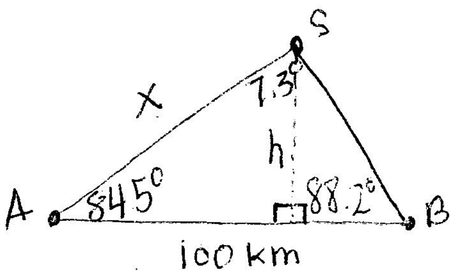
Ex7: Solve ΔABC: $\angle C = 82^\circ$, $c = 12$, $b = 15$



SSA
acute
12 < 15
case 3

$15\sin 82^\circ = 14.85$
$12 < 14.85$
no soln

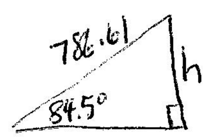
Ex 8: A satellite passes over two tracking stations, *A* and *B*, 100 km apart. When the satellite is between the two stations the angle of elevation at the stations are measured as 84.5° and 88.2° respectively. What is the distance between the satellite and station *A.?* How high is the satellite ~~of~~ off the ground?



ASA

$$\frac{\sin 88.2^\circ}{x} = \frac{\sin 7.3^\circ}{100}$$

$$x = \frac{100\sin 88.2^\circ}{\sin 7.3^\circ} = 786.61 \text{ km}$$



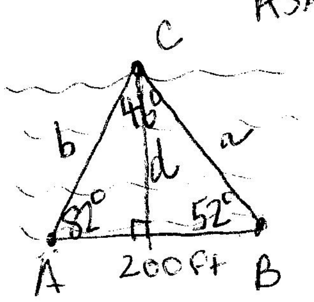
$$\sin 84.5^\circ = \frac{h}{786.61}$$

$$h = 786.61\sin 84.5^\circ$$

$$h = 782.99 \text{ km}$$

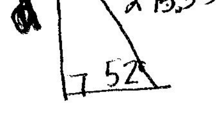
Ex 9: To find the distance across a river, a surveyor chooses point *A* and *B,* which are 200 ft. apart on one side of the river. She chooses a reference point *C* on the opposite side of the river and finds that $< BAC = 82^\circ$ and $< ABC = 52^\circ$. Find the distance across the river.

ASA



$$\frac{\sin 82^\circ}{a} = \frac{\sin 46^\circ}{200}$$

$$a = \frac{200\sin 82^\circ}{\sin 46^\circ} = 275.33$$



$$\sin 52^\circ = \frac{d}{275.33}$$

$$d = 216.96 \text{ ft}$$