Notes--Limits Approaching ∞ or $-\infty$

end behavior

Review Finding horizontal asymptotes . . .

$$f(x) = \frac{ax^n + \ldots}{bx^m + \ldots}$$ ← nth degree polynomial; ← mth degree polynomial

1. If $n < m$, then the x-axis is the horizontal asymptote. $y = 0$
2. If $n = m$, then the horizontal asymptote is the line $y = \frac{a}{b}$
3. If $n > m$, then there is no horizontal asymptote. (There is a slant diagonal or oblique asymptote.)

Example 1 Find each limit.

1. $\lim_{x\to\infty}\left(\frac{x^2+1}{x^3+2x}\right) = 0$

2. $\lim_{x\to-\infty}\left(\frac{3x+1}{x^2}\right) = 0$

3. $\lim_{x\to\infty}\left(\frac{5x^2+2x-3}{9x^2+x+4}\right) = \frac{5}{9}$

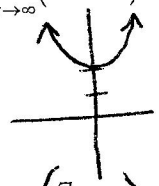
4. $\lim_{x\to\infty}\left(\frac{4x^2+1}{2x}\right)$ = DNE, direction of ∞

   $\frac{4x^2}{2x} = 2x$

5. $\lim_{x\to-\infty}\left(\frac{4x^2+1}{2x}\right)$ = DNE, direction of $-\infty$

6. $\lim_{x\to\infty}\frac{(2x+1)(x-7)}{(3x-2)(4x+1)} = \lim_{x\to\infty}\frac{2x^2-13x-7}{12x^2-5x-2} = \frac{2}{12} = \boxed{\frac{1}{6}}$

7. $\lim_{x\to\infty}(2+x^2) = \infty$



8. $\lim_{x\to\infty}\left(\frac{7}{x^2}+5\right) = \lim_{x\to a}\frac{7}{x^2} + \lim_{x\to\infty}5 = 0+5 = \boxed{5}$

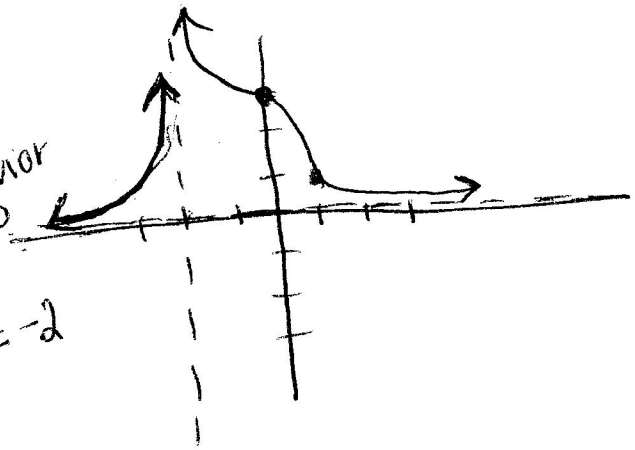
Example 2 Draw a function if . . .

$f(0)=3$ (0,3)

$f(1)=1$ (1,1)

$\lim_{x\to\infty} f(x)=0$, $\lim_{x\to-\infty} f(x)=0$ } end behavior h.a. $y=0$

$\lim_{x\to-2^+} f(x)=\infty$, $\lim_{x\to-2^-} f(x)=\infty$ } V.a. $x=-2$



✓ $f(0)=0$ (0,0)

✓ *there is a removeable discontinuity at* $x=4$ hole

✓ $\lim_{x\to\infty} f(x)=1$, ✓ $\lim_{x\to-\infty} f(x)=-1$ } end behavior

✓ $\lim_{x\to4} f(x)=2$

✓ $\lim_{x\to-3} f(x)=$ *does not exist* (jump OR V.a.)