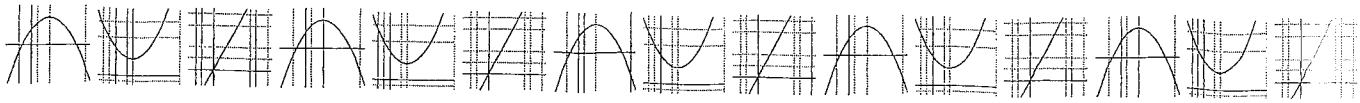


Pre-Calculus

Name: _____

Notes (2.1)---Review Linear & Quadratic Polynomial Functions

Read the definition of a polynomial:

Sometimes it is actually easier to look for what makes a function NOT a polynomial functions rather than the other way around.

DEFINITION

Let n be a nonnegative integer (exponent) and let $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ be real numbers (coefficients) with $a_n \neq 0$. The function given by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

is a **polynomial function of degree** n. The **leading coefficient** is a_n.

The zero function $f(x) = 0$ is a polynomial function. It has no degree and no leading coefficient.

List some things to look for that prove a function is NOT a polynomial: fractional exponents, imaginary coefficients, negative exponents

Ex1) Determine whether each of the following is a polynomial function, if it is not state why it isn't:

a) $f(x) = 5x^{-1} = \frac{5}{x}$ — no – neg exponent

b) $g(x) = 4x^2 + ex - 10$ — yes

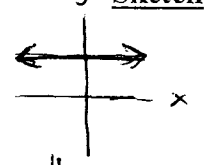
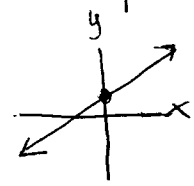
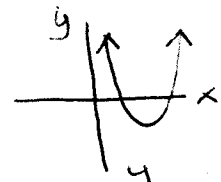
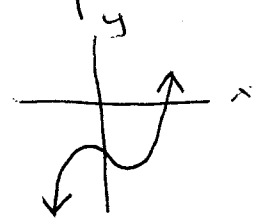
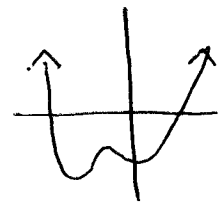
c) $h(x) = -3x^{\pi} + 4x^3 + 11x$ — no – fractional exponent

NOW YOU TRY☺

d) $j(x) = 2x^{1/2} + 10x + 6$ — no - fractional exponent

e) $k(x) = \frac{1}{4}x^4 - 9x^3 + 7$ — yes

f) $f(x) = -x^4 + 5x^3 - 8ix^2 + 7$ — no - imaginary coefficient

	Name	Example	Sketch
Polynomial of Degree = 0 → Called a "constant" function → $f(x) =$		4	
Polynomial of Degree = 1 → Called a "linear" function → $f(x) =$		$\frac{1}{2}x+1$	
Polynomial of Degree = 2 → Called a "quadratic" function → $f(x) =$		x^2-5x+4	
Polynomial of Degree = 3 → Called a "cubic" function → $f(x) =$		x^3+x-2	
Polynomial of Degree = 4 → Called a "quartic" function → $f(x) =$		$x^4+2x^3+3x^2-x-2$	

****There are two polynomial functions you are expected to be very familiar with… Linear & Quadratic****

LINEAR		***QUADRATIC***	
Slope-Intercept Form:	$y = mx + b$	Vertex Form:	$y = a(x-h)^2 + k$ (h, k) vertex $x = h$ axis of symm.
Point-Slope Form:	$y - y_1 = m(x - x_1)$	Standard Form:	$y = ax^2 + bx + c$
Slope Equation:	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$	Finding the Vertex:	(h, k) $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
Standard Form:	$Ax + By = C$ no fractions "A" positive	Completing the Square:	puts the eqn into vertex form
		Ways to solve quadratics:	quad. formula, factor, x-intercepts

Ex.2) Write a linear function that satisfies all of the following conditions.

a) through: (0, 3) and (−4, −1)

$m = \frac{-1-3}{-4-0} = \frac{-4}{-4} = 1$

$b = 3$ $\boxed{y = x + 3}$

b) through: (0, 2) and (1, −3)

$b = 2$

$m = \frac{-3-2}{1-0} = \frac{-5}{1} = -5$ $\boxed{y = -5x + 2}$

c) f(−4)= 2 and f(0)= −5

$(-4, 2)$ $(0, -5)$ $b = -5$

$m = \frac{-5-2}{0-(-4)} = \frac{-7}{4}$ $\boxed{y = -\frac{7}{4}x - 5}$

d) f(4)= −2 and f(−4)= −4

points: $(4, -2)$ $(-4, -4)$

$m = \frac{-4 - -2}{-4-4} = \frac{-2}{-8} = \frac{1}{4}$

$y - -2 = \frac{1}{4}(x - 4)$

$y + 2 = \frac{1}{4}(x - 4)$

$y + 2 = \frac{1}{4}x - 1$

$\boxed{y = \frac{1}{4}x - 3}$

Ex.3) Write an equation for the quadratic function with the given vertex and point.

$y = a(x-h)^2 + k$

a) Vertex (0, 0) passing through (−2, 8)

h k x y

$8 = a(-2-0)^2 + 0$

$8 = 4a$

$a = 2$

$\boxed{y = 2(x-0)^2 + 0 = 2x^2}$

b) Vertex (2, 0) passing through (1, 3).

h k x y

$3 = a(1-2)^2 + 0$

$3 = a$

$\boxed{y = 3(x-2)^2 + 0}$

c)) Vertex (−3, 0) passing through (−5, −4).

d) Vertex (−3, 4) passing through (0, 0)

h k x y

$0 = a(0 - -3)^2 + 4$

$0 = 9a + 4$

$-4 = 9a$

$a = -4/9$

$y = -\frac{4}{9}(x - -3)^2 + 4$

$\boxed{y = -\frac{4}{9}(x+3)^2 + 4}$

Ex.4) Use completing the square to write the following equations in vertex form.

a) $y = x^2 + 6x - 11$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$y = x^2 + 6x + 9 - 11 - 9$

$y = (x+3)(x+3) - 20$

$\boxed{y = (x+3)^2 - 20}$

b) $y = 2x^2 - 12x + 1$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$y = 2x^2 - 12x + 1$

$y = 2(x^2 - 6x + 9) + 1 - 18$

$\boxed{y = 2(x-3)^2 - 17}$

c) $y = -x^2 - 3x - 5$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$y = -x^2 - 3x - 5$

$y = -1\left(x^2 + 3x + \frac{9}{4}\right) - 5 + \frac{9}{4}$

$\boxed{y = -1\left(x + \frac{3}{2}\right)^2 - \frac{11}{4}}$

d) $y = \frac{1}{3}x^2 - 4x - 1$

$\left(\frac{-12}{2}\right)^2 = 36$

$y = \frac{1}{3}x^2 - 4x + 1$

$y = \frac{1}{3}(x^2 - 12x + 36) - 1 - 12$

$\boxed{y = \frac{1}{3}(x-6)^2 - 13}$

STEPS

1. separate var. from constant
2. coeff of $x^2 \Rightarrow 1$ (factor if not)
3. $\left(\frac{\text{coeff. of linear term}}{2}\right)^2$ add that # & compensate
4. factor / arithmetic