

Notes (Section 5.6)-The Law of Cosines

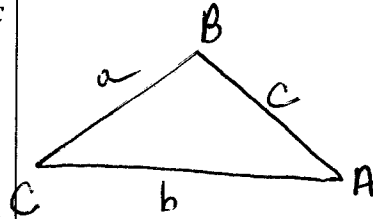
The Law of Cosines states the ratio of the sine of an angle to the length its opposite angle is the same for all three angles.

In any $\triangle ABC$ with angles A, B , and C opposite sides a, b , and c respectively, the following equation is true:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



We can use the Law of Cosines to solve triangles when given SSS & SAS.

Ex 1: Solve $\triangle ABC$: $a = 10, b = 11, c = 17$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$17^2 = 10^2 + 11^2 - 2(10)(11) \cos C$$

$$289 = 100 + 121 - 220 \cos C$$

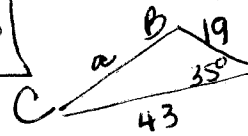
$$68 = -220 \cos C$$

$$-\frac{17}{55} = \cos C$$

$$C = \cos^{-1}\left(-\frac{17}{55}\right) = 108.0^\circ$$

$$\begin{aligned} A &= 34.0^\circ \\ B &= 38.0^\circ \\ C &= 108.0^\circ \end{aligned}$$

Ex 2: Solve $\triangle ABC$: $A = 35^\circ, b = 43, c = 19$



$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$43^2 = 29.52^2 + 19^2 - 2(29.52)(19) \cos B$$

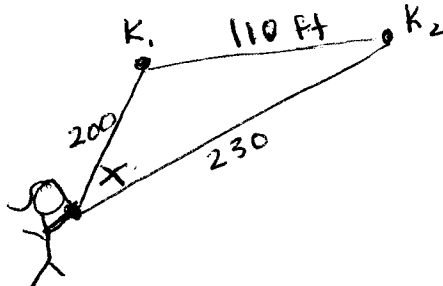
$$-0.5495778423 = \cos B$$

$$B = 123.3^\circ$$

$$C = 21.7^\circ$$

$$\begin{aligned} a &= 29.52 \\ B &= 123.3^\circ \\ C &= 21.7^\circ \end{aligned}$$

Ex 3: A girl is flying two kites at the same time. If the strings are 200 ft and 230 ft long and the kites are 110 ft apart, what angle do the strings make in her hand?



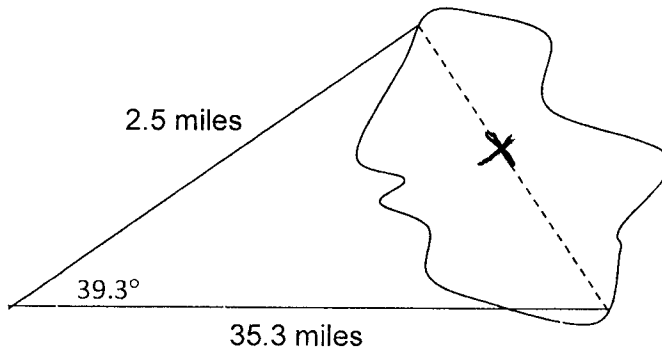
$$110^2 = 200^2 + 230^2 - 2(200)(230) \cos X$$

$$12100 = 40000 + 52900 - 92000 \cos X$$

$$-80800 = -92000 \cos X$$

$$\frac{101}{115} = \cos X \quad X = \cos^{-1}\left(\frac{101}{115}\right) = 28.6^\circ$$

Ex 4: To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake using this information.



$$X^2 = 2.5^2 + 35.3^2 - 2(2.5)(35.3) \cos 39.3^\circ$$

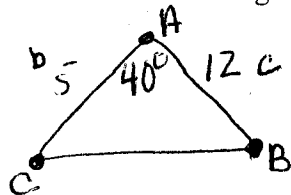
$$X^2 = 1115.757203$$

$$X = 33.40 \text{ miles}$$

Area of a triangle

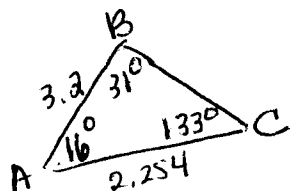
$$\Delta \text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

Ex1) Find the area of a triangle with sides of length 5 and 12 and included angle 40° .



$$\text{area} = \frac{1}{2}(5)(12)\sin 40^\circ = 19.28 \text{ units}^2$$

Ex2) Find the area of Triangle ABC to the nearest thousandth if $c = 3.2$, $A = 16^\circ$, $B = 31^\circ$



$$\frac{\sin 31^\circ}{b} = \frac{\sin 133^\circ}{3.2}$$

$$b = \frac{3.2 \sin 31^\circ}{\sin 133^\circ} = 2.253522251$$

$$\text{area} = \frac{1}{2}(3.2)(2.254)\sin 16^\circ = 0.994 \text{ units}^2$$

Theorem Heron's Formula

Let a , b and c be the sides of ΔABC , let s denote the **semi-perimeter**; $s = \frac{(a+b+c)}{2}$.

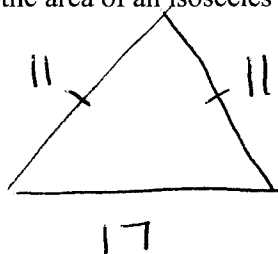
Then the area of ΔABC is given by $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

Ex 3) Find the area of triangle ABC with $a = 2$, $b = 7$, $c = 8$

$$s = \frac{2+7+8}{2} = 8.5$$

$$\text{area} = \sqrt{8.5(8.5-2)(8.5-7)(8.5-8)} = 6.44 \text{ units}^2$$

Ex 4) Find the area of an isosceles triangle with a perimeter of 39 and a base of length 17 inches.



$$39 - 17 = 22$$

$$s = \frac{39}{2} = 19.5$$

$$\text{area} = \sqrt{19.5(19.5-11)(19.5-11)(19.5-17)} = 59.35 \text{ in}^2$$