NOTES--PROPERTIES OF LOGARITHMS

Logarithmic Properties	
Product Rule	$\log_a(xy) = \log_a x + \log_a y$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power Rule	$\log_a x^p = p\log_a x$
Change of Base Rule	$\log_a x = \frac{\log_b x}{\log_b a}$
Equality Rule	If $\log_a x = \log_a y$ then $x = y$

Example 1 Write as a sum or difference without powers.

A. $\log_a\left(x\sqrt{x^2+1}\right)$

$\log_a x + \log_a \sqrt{x^2+1}$

$\log_a (x^2+1)^{\frac{1}{2}}$

$\log_a x + \frac{1}{2}\log_a (x^2+1)$

B. $\log_4 \frac{x^4}{(x-1)^3}$

$\log_4 x^4 - \log_4 (x-1)^3$

$4\log_4 x - 3\log_4 (x-1)$

C. $\log_4 \frac{x^2\sqrt{x^2+1}}{(x+1)^3}$

$\log_4 x^2 + \log_4 \sqrt{x^2+1} - \log_4 (x+1)^3$

$2\log_4 x + \frac{1}{2}\log_4 (x^2+1) - 3\log_4 (x+1)$

Example 2 Write as a single log: $4\log_3 x + \frac{1}{2}\log_3 (x+1)$

$\log_3 x^4 + \log_3 (x+1)^{1/2}$

$\log_3 \left(x^4 \cdot \sqrt{x+1}\right)$

Example 3 Evaluate $\log_3 7$ using the change of base formula.

$$\frac{\log_{10} 7}{\log_{10} 3} = 1.771 \qquad \frac{\ln 7}{\ln 3}$$

Example 4 Solve.

A. $\log_3(4x-7)=2$

$3^2 = 4x-7$

$9 = 4x-7$

$16 = 4x$

$4 = x$

B. $\log_4(x+3)+\log_4(2-x)=1$

$\log_4[(x+3)(2-x)] = 1$

$4^1 = 2x - x^2 + 6 - 3x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x+2=0 \quad x-1=0$

$x = -2, \; x = 1$

C. $3\log_4 x = 7\log_4 2$

$\log_4 x^3 = \log_4 2^7$

$x^3 = 2^7$

$x^3 = 128$

$x = \sqrt[3]{128}$

D. $3^{4x+1} = 27$

$3^{4x+1} = 3^3$

$4x+1 = 3$

$4x = 2$

$x = \frac{2}{4} = \frac{1}{2}$

E. $3^{x-1} = 7^{4-3x}$

$\log 3^{x-1} = \log 7^{4-3x}$

$(x-1)\log 3 = (4-3x)\log 7$

$x\log 3 - \log 3 = 4\log 7 - 3x\cdot\log 7$

$x\log 3 + 3x\log 7 = 4\log 7 + \log 3$

$x(\log 3 + 3\log 7) = 4\log 7 + \log 3$

$$x = \frac{4\log 7 + \log 3}{\log 3 + 3\log 7}$$

$$\frac{\log 2401 + \log 3}{\log 3 + \log 343} = \frac{\log 7203}{\log 1029} = \log_{1029} 7203$$

F. $\frac{20e^{0.3x}}{20} = \frac{60}{20}$

$e^{0.3x} = 3$

$\ln e^{0.3x} = \ln 3$

$0.3x \cdot \ln e = \ln 3$

$\frac{0.3x}{0.3} = \frac{\ln 3}{0.3}$

$x = 3.662$