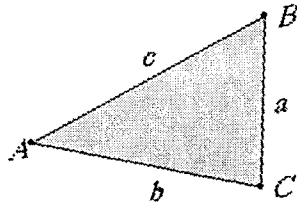


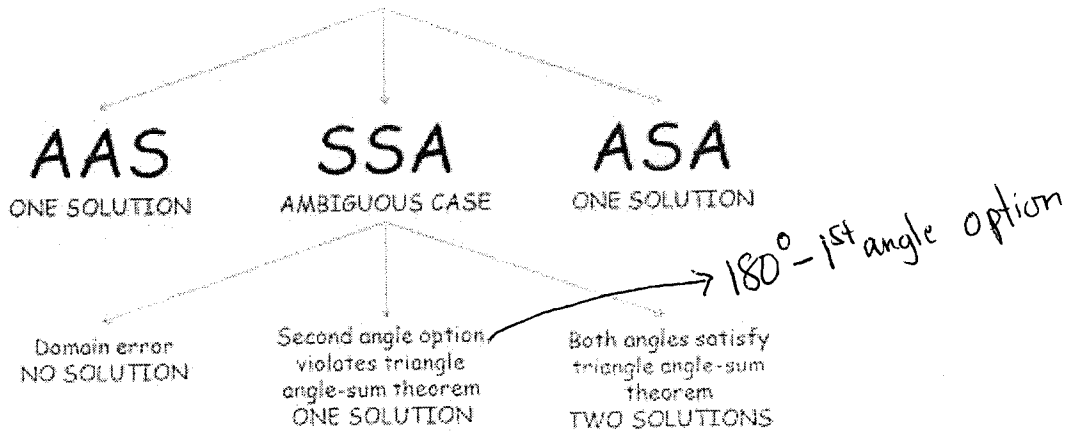
NOTES--Law of Sines and Law of Cosines

Law of Sines

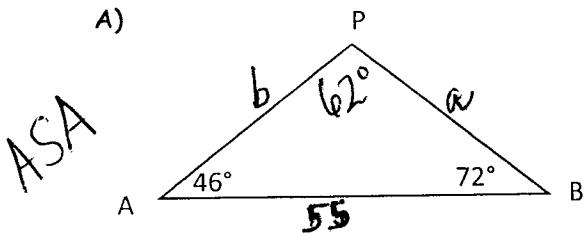


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Sines



Example 1 Solve the triangle.



$$P = 180^\circ - 46^\circ - 72^\circ = 62^\circ$$

$$\frac{\sin 72^\circ}{b} = \frac{\sin 62^\circ}{55}$$

$$b \cdot \sin 62^\circ = 55 \sin 72^\circ$$

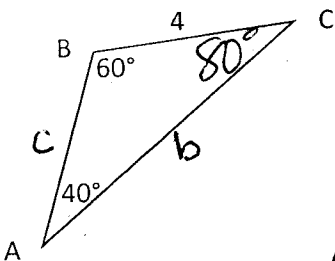
$$b = \frac{55 \sin 72^\circ}{\sin 62^\circ} = 59.2$$

$$\frac{\sin 46^\circ}{a} = \frac{\sin 62^\circ}{55}$$

$$a \cdot \sin 62^\circ = 55 \sin 46^\circ$$

$$a = \frac{55 \sin 46^\circ}{\sin 62^\circ} = 44.8$$

AAS



$$C = 180^\circ - 60^\circ - 40^\circ = 80^\circ$$

$$\frac{\sin 80^\circ}{c} = \frac{\sin 40^\circ}{4}$$

$$c \cdot \sin 40^\circ = 4 \sin 80^\circ$$

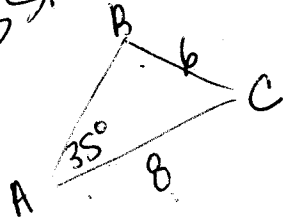
$$c = \frac{4 \sin 80^\circ}{\sin 40^\circ} = 6.1$$

$$\frac{\sin 60^\circ}{b} = \frac{\sin 40^\circ}{4}$$

$$b \cdot \sin 40^\circ = 4 \sin 60^\circ$$

$$b = \frac{4 \sin 60^\circ}{\sin 40^\circ} = 5.4 \quad 5$$

SSA c) $a=6, b=8, A=35^\circ$



$$\frac{\sin B}{8} = \frac{\sin 35^\circ}{6}$$

$$6 \sin B = 8 \sin 35^\circ$$

$$B = \sin^{-1}\left(\frac{8 \sin 35^\circ}{6}\right) = 49.9^\circ \text{ or } 130.1^\circ$$

$$C = 180^\circ - 35^\circ - 49.9^\circ = 95.1^\circ$$

$$\frac{\sin 95.1^\circ}{c} = \frac{\sin 35^\circ}{6}$$

$$c \cdot \sin 35^\circ = 6 \sin 95.1^\circ$$

$$c = \frac{6 \sin 95.1^\circ}{\sin 35^\circ} = 10.4$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\begin{array}{l|l} B=49.9^\circ & B=130.1^\circ \\ C=95.1^\circ & C=14.9^\circ \\ c=10.4 & c=2.7 \end{array}$$

$$C = 180^\circ - 35^\circ - 130.1^\circ = 14.9^\circ$$

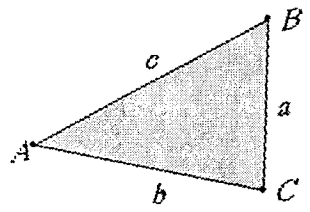
$$\frac{\sin 14.9^\circ}{c} = \frac{\sin 35^\circ}{6}$$

$$c \sin 35^\circ = 6 \sin 14.9^\circ$$

$$c = \frac{6 \sin 14.9^\circ}{\sin 35^\circ}$$

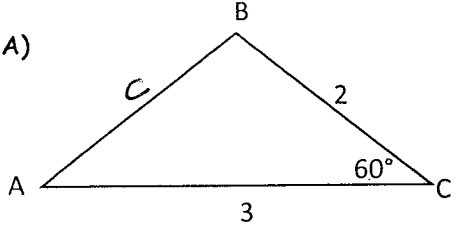
$$c = 2.7$$

Law of Cosines



Example 2

SAS A)



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 2^2 + 3^2 - 2(2)(3) \cos 60^\circ$$

$$\begin{aligned} c^2 &= 7 \\ c &= \sqrt{7} \end{aligned}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$3^2 = 2^2 + \sqrt{7}^2 - 2(2)(\sqrt{7}) \cos B$$

$$-2 = -4\sqrt{7} \cos B$$

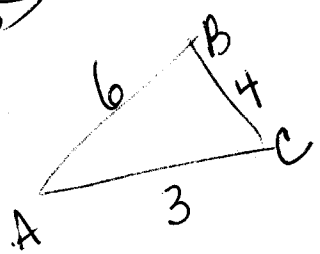
$$\cos B = \frac{-2}{-4\sqrt{7}}$$

$$B = \cos^{-1}\left(\frac{2}{4\sqrt{7}}\right) = 79.1^\circ$$

$$A = 180^\circ - 60^\circ - 79.1^\circ = 40.9^\circ$$

SSS B)

$a=4, b=3, c=6$



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$6^2 = 4^2 + 3^2 - 2(4)(3) \cos C$$

$$11 = -24 \cos C$$

$$\frac{11}{-24} = \cos C$$

$$C = \cos^{-1}\left(\frac{11}{-24}\right) = 117.3^\circ$$

$$\frac{\sin A}{4} = \frac{\sin 117.3^\circ}{6}$$

$$6 \sin A = 4 \sin 117.3^\circ$$

$$A = \sin^{-1}\left(\frac{4 \sin 117.3^\circ}{6}\right)$$

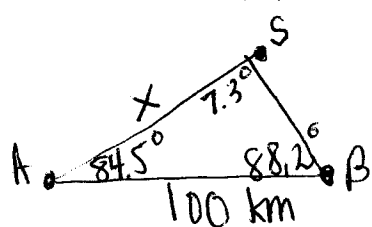
$$A = 36.3^\circ$$

$$B = 180^\circ - 36.3^\circ - 117.3^\circ = 26.4^\circ$$

6

Example 3

A satellite passes over two tracking stations, A and B, 100 km apart. When the satellite is between the two stations the angle of elevation at the stations are measured as 84.5° and 88.2° respectively. What is the distance between the satellite and station A?



$$S = 180^\circ - 84.5^\circ - 88.2^\circ = 7.3^\circ$$

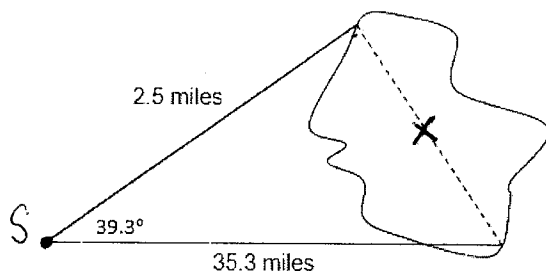
$$\frac{\sin 88.2^\circ}{x} = \frac{\sin 7.3^\circ}{100}$$

$$x = \frac{100 \sin 88.2^\circ}{\sin 7.3^\circ}$$

$$x = \boxed{786.6 \text{ km}}$$

Example 4

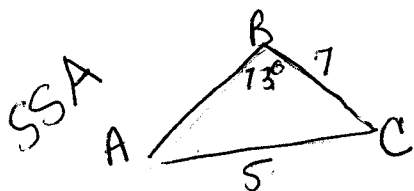
To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake using this information.



$$x^2 = 2.5^2 + 35.3^2 - 2(2.5)(35.3)\cos 39.3^\circ$$
$$x = \boxed{33.4 \text{ miles}}$$

Example 5

Solve the triangle: $m\angle B = 73^\circ$, $a = 7$, $b = 5$



$$\frac{\sin A}{7} = \frac{\sin 73^\circ}{5}$$

$$5 \cdot \sin A = 7 \sin 73^\circ$$

$$\sin A = \frac{7 \sin 73^\circ}{5}$$

$$A = \sin^{-1}(\uparrow)$$

$\boxed{\text{no soln.}}$