Area and Volume

AREA

if you have functions of x, use $A = \int_{x_1}^{x_2} [(\text{top curve}) - (\text{bottom curve})]\, dx$

if you have functions of y, use $A = \int_{y_1}^{y_2} [(\text{right curve}) - (\text{left curve})]\, dy$

VOLUME

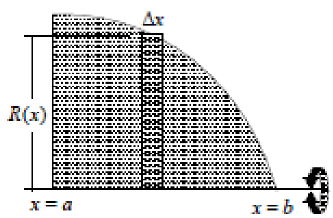
The Disk Method

Horizontal Axis of Revolution

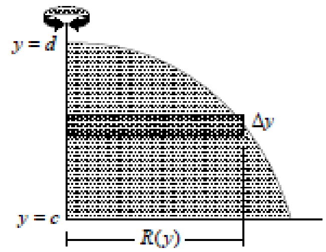
$$V = \pi \int_a^b [R(x)]^2 \, dx$$

Vertical Axis of Revolution

$$V = \pi \int_c^d [R(y)]^2 \, dy$$



Horizontal axis of revolution



Vertical axis of revolution

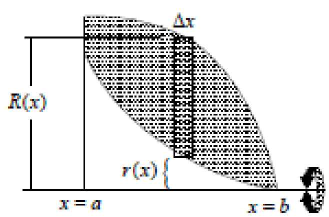
The Washer Method

Horizontal Axis of Revolution

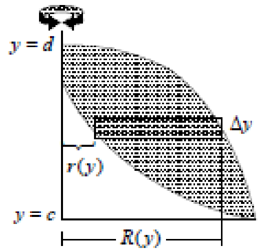
$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\, dx$$

Vertical Axis of Revolution

$$V = \pi \int_c^d ([R(y)]^2 - [r(y)]^2)\, dy$$



Horizontal axis of revolution



Vertical axis of revolution

Volume using Cross-sections

The volume of a solid of known cross-sectional area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b.

1. For cross sections of area $A(x)$ taken perpendicular to the x-axis, the volume is

$$V = \int_a^b A(x)\,dx\,.$$

2. For cross sections of area $A(y)$ taken perpendicular to the y-axis, the volume is

$$V = \int_c^d A(y)\,dy\,.$$

Shell method

$$V = \int_a^b 2\pi x f(x)\,dx = 2\pi \int_a^b x f(x)\,dx$$

Horizontal Axis of Revolution

$$\text{Volume} = V = 2\pi \int_c^d p(y)h(y)\,dy$$

Vertical Axis of Revolution

$$\text{Volume} = V = 2\pi \int_a^b p(x)h(x)\,dx$$