<u>Integration Techniques</u>

<u>PARTIAL FRACTIONS</u>—used to integrate rational expressions (when other AB techniques do not work)

Steps:
1) Make sure the degree of the numerator is less than the degree of the denominator. If it isn't, divide first.
2) Factor the denominator completely.
3) Write the rational expression as a sum of fractions:

   $$\text{original rational expression} = \frac{A}{1st\ \ factor} + \frac{B}{2nd\ \ factor} + \frac{C}{3rd\ \ factor} + \ldots$$

4) Determine the values of A, B, C, etc. by clearing the fractions (multiply both sides of the equation by the denominator of the original rational expression) and by choosing "smart" values of x to substitute into the equation. A "smart" value is the zero of each linear factor.
5) Integrate each fraction. Remember to use u-substitution if a factor in the denominator has a coefficient other than 1 for x.

<u>INTEGRATION BY PARTS</u>—used to integrate a log function, an inverse trig function, or a product of two different types of functions of x (when other AB techniques do not work)

$$\int u\, dv = uv - \int v\, du$$

u differentiates to zero (usually)
dv is easy to integrate

Choose u in this order: **LIATE**
Log, **I**nverse trig, **A**lgebraic, **T**rigonometric, **E**xponential

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<u>IMPROPER INTEGRATION</u>—used when a bound (or both) is infinite or when the interval is discontinuous

Steps:
1) Replace the infinite bound or discontinuity with a variable and change the problem to a limit approaching infinity or the discontinuity.
2) Find the antiderivative and evaluate the definite integral.
3) Evaluate the limit of the expression. If the limit exists $\Rightarrow$ the integral converges. If the limit does not exist $\Rightarrow$ the integral diverges.

Improper Integrals with Infinite Integration Limits

1. If $f(x)$ is continuous on $[a,\infty)$, then

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx\ .$$

2. If $f(x)$ is continuous on $(-\infty,b]$, then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx\ .$$

3. If $f(x)$ is continuous on $(-\infty,\infty)$, then

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx\ ,$$

where c is any real number.

Improper Integrals with Infinite Discontinuities

1. If $f(x)$ is continuous on $[a,b)$ and has an infinite discontinuity at b, then

$$\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx\ .$$

2. If $f(x)$ is continuous on $(a,b]$ and has an infinite discontinuity at a, then

$$\int_a^b f(x)\,dx = \lim_{c\to a^+}\int_c^b f(x)\,dx\ .$$

3. If $f(x)$ is continuous on $[a,b]$, except for some number c in (a,b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx\ ,$$

where c is any real number.