Series (for 2020 AP Exam)

Tests You Need to Know

Geometric Series Test:

If the series has the form $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^{n}$, then the series converges if $|r|<1$ and diverges otherwise. If the series converges, then it converges to $\frac{a_1}{1-r}$.

***p*-series test:**

If the series has the form $\sum \frac{1}{n^p}$, then the series converges if $p>1$ and diverges otherwise. When $p=1$, the series is the divergent Harmonic series.

Alternating Series Test:

If the series has the form $\sum(-1)^n a_n$, then the series converges if $0<a_{n+1}\le a_n$ (decreasing terms) for all n, for some n, and $\lim_{n\to\infty} b_n = 0$. If either of these conditions fails, the test fails, and you need use a different test.

Ratio Test:

If $a_n>0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = N$ (where N is a real number), then

1. $\sum a_n$ converges absolutely (and hence converges) if $N<1$
2. $\sum a_n$ diverges if $N>1$ or $N=\infty$
3. The test is inconclusive if $N=1$ (use another test)

Use this test for series whose terms converge rapidly, for instance those involving exponentials and/or factorials!!!!!!

Taylor Polynomial

The Taylor polynomial of degree n centered at $x=a$ is

$$f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\ldots+\frac{f^{(n)}(a)}{n!}(x-a)^n$$

key ideas:

- The coefficient of the n$^{\text{th}}$ degree term centered at $x=a$ is $\frac{f^{(n)}(a)}{n!}$.
- Generally, as the degree of the polynomial increases, the polynomial more closely approximates the function over some interval.
- A Taylor polynomial centered at $x=a$ can be used to approximate function values of f near $x=a$.