BC Calculus Series Practice

1.

The sum of the geometric series $\frac{2}{21}+\frac{4}{63}+\frac{8}{189}+\ldots$ is

(A) $\frac{5}{21}$ (B) $\frac{2}{7}$ (C) $\frac{4}{7}$ (D) The series diverges

2.

$\sum_{n=1}^{\infty}\frac{(3)^{n+1}}{5^n}=$

(A) $\frac{3}{5}$ (B) $\frac{5}{2}$ (C) $\frac{9}{2}$ (D) The series diverges

3.

What are all values of p for which $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^p+1}$ converges?

(A) $p>0$ (B) $p>\frac{1}{2}$ (C) $p>1$ (D) $p>\frac{3}{2}$

4.

What are all values of k for which the series $1+(\sqrt{2})^k+(\sqrt{3})^k+(\sqrt{4})^k+\cdots+(\sqrt{n})^k+\cdots$ converges?

(A) $k<-2$ (B) $k<-1$ (C) $k>1$ (D) $k>2$

5.

Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n}}{n}$ II. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\ln n}$ III. $\sum_{n=1}^{\infty}\cos(n\pi)$

(A) I only (B) II only (C) III only (D) I and II only

6.

Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^n \cos(\frac{\pi}{n})$ II. $\sum_{n=1}^{\infty} \sin(\frac{2n-1}{2})\pi$ III. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n^2+1}$

(A) I only (B) II only (C) III only (D) I and II only

7.

For what integer k, $k>1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{n^2\sqrt{n}}{n^k+1}$ converge?

(A) 3 (B) 4 (C) 5 (D) 6

8.

Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ II. $\sum_{n=1}^{\infty} \frac{n}{3^n}$ III. $\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$

(A) I only (B) II only (C) II and III only (D) I, II, and III

9.

Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ II. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ III. $\sum_{n=1}^{\infty} \frac{n^9}{9^n}$

(A) I only (B) II only (C) I and II only (D) I, II, and III

10.

Let $P(x)=\frac{1}{3}-\frac{2}{3}x+\frac{2}{3}x^2-\frac{4}{9}x^3+\frac{2}{9}x^4$ be the fourth-degree Taylor polynomial for the function f about $x=0$. What is the value of $f^{(4)}(0)$?

(A) $-\frac{32}{3}$ (B) $-\frac{4}{3}$ (C) $\frac{8}{9}$ (D) $\frac{16}{3}$

11.

Let $P(x) = 4 - 3x^2 + \frac{13}{12}x^4 - \frac{121}{360}x^6$ be the sixth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(A) $-\frac{121}{15}$ (B) $-\frac{3}{2}$ (C) 0 (D) $\frac{121}{15}$

12.

Let f be a function that has derivatives of all orders for all real numbers. If $f(1) = 2$, $f'(1) = -3$, $f''(1) = 4$, and $f'''(1) = -9$, which of the following is the third-degree Taylor polynomial for f about $x = 1$?

(A) $P(x) = 2 - 3(x-1) + 2(x-1)^2 - \frac{3}{2}(x-1)^3$

(B) $P(x) = 2 - 3(x+1) + 2(x+1)^2 - \frac{3}{2}(x+1)^3$

(C) $P(x) = 2 - 3(x-1) + 4(x-1)^2 - 9(x-1)^3$

(D) $P(x) = 2 - 3(x+1) + 2(x+1)^2 - 3(x+1)^3$

13.

The third-degree Taylor polynomial of xe^x about $x = 0$ is

(A) $P_3(x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3$

(B) $P_3(x) = x + x^2 + \frac{1}{2}x^3$

(C) $P_3(x) = x + x^2 - \frac{1}{3}x^3$

(D) $P_3(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$

14.

The second-degree Taylor polynomial of $\sec x$ about $x = \frac{\pi}{4}$ is

(A) $P_2(x) = 1 + \sqrt{2}(x - \frac{\pi}{4}) + \sqrt{2}(x - \frac{\pi}{4})^2$

(B) $P_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) + \frac{3\sqrt{2}}{3!}(x - \frac{\pi}{4})^2$

(C) $P_2(x) = \sqrt{2} + \sqrt{2}(x - \frac{\pi}{4}) + \frac{3\sqrt{2}}{2!}(x - \frac{\pi}{4})^2$

(D) $P_2(x) = 1 + \sqrt{2}(x - \frac{\pi}{4}) + \frac{3\sqrt{2}}{3!}(x - \frac{\pi}{4})^2$

15.

A function f has derivatives of all orders at $x = 0$. Let P_n denote the nth- degree Taylor polynomial for f about $x = 0$. It is known that $f(0) = \frac{1}{3}$ and $f''(0) = \frac{4}{3}$. If $P_2(\frac{1}{2}) = \frac{1}{8}$, what is the value of $f'(0)$?

(A) $-\frac{3}{8}$ (B) $-\frac{3}{4}$ (C) $-\frac{5}{4}$ (D) $-\frac{3}{2}$
