Curve Analysis and Optimization

Using the 1^{st} and 2^{nd} Derivatives

1^{st} derivative $\Rightarrow$ max/min, increasing/decreasing, slope of the tangent line to the curve
2^{nd} derivative $\Rightarrow$ inflection points, concavity

Properties of the 1^{st} Derivative
increasing: slopes of tangent lines are positive; where $f'(x) > 0$

decreasing: slopes of tangent lines are negative; where $f'(x) < 0$

maximum point: slopes switch from positive to negative
minimum point: slopes switch from negative to positive

To find relative max/min, increasing/decreasing . . .

1) Find the 1^{st} derivative.
2) Find the critical values (where the 1^{st} derivative undefined and/or equal to 0)
3) Plug #'s into the 1^{st} derivative within the intervals on the number line.

    $f'(x) > 0$ means the graph is increasing on that interval

    $f'(x) < 0$ means the graph is decreasing on that interval

    $f'(x)$ switches from $+$ to $-$ means a relative max occurs at that x-value

    $f'(x)$ switches from $-$ to $+$ means a relative min occurs at that x-value

    *to find the y-value of the max/min, plug into the *original equation*

Properties of the 2^{nd} Derivative
concave up: slopes of tangent lines are increasing; where $f''(x) > 0$

concave down: slopes of tangent lines are decreasing; where $f''(x) < 0$

inflection points: points where the graph switches concavity (where slopes of the tangent lines change from increasing to decreasing or vice versa)

To find concavity and inflection points . . .

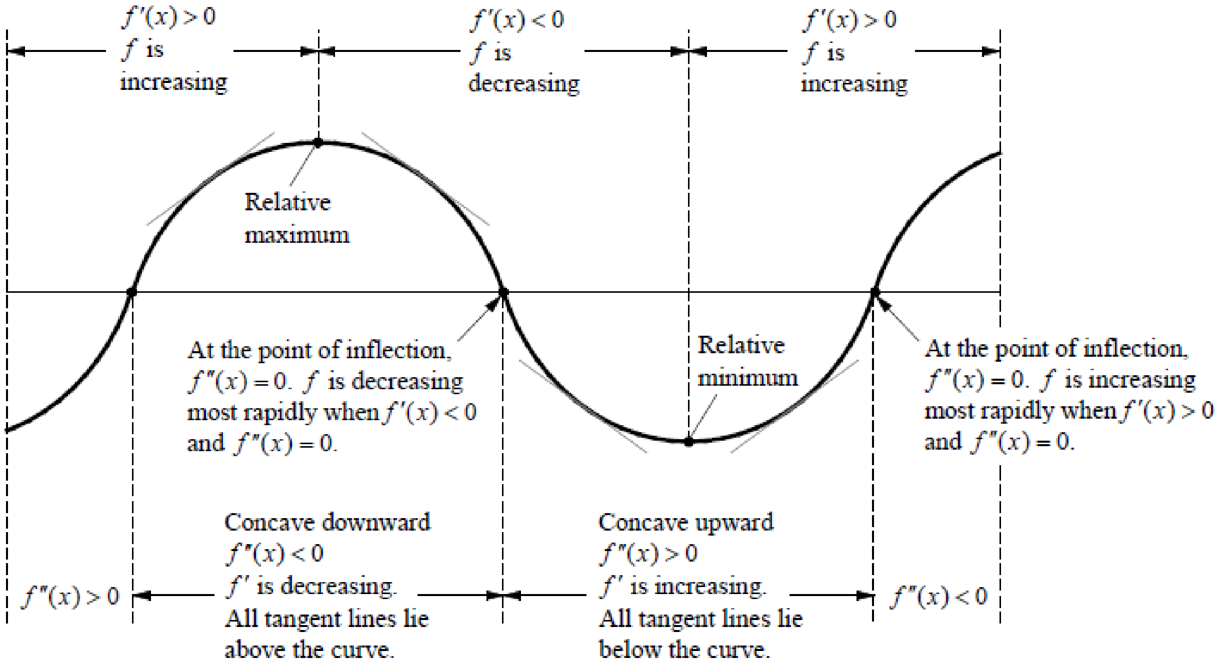
1) Find the 2^{nd} derivative.
2) Find the values where the 2^{nd} derivative is undefined and/or equal to 0
3) Plug #'s into the 2^{nd} derivative within the intervals on the number line.

    $f''(x) > 0$ means the graph is concave up on that interval

    $f''(x) < 0$ means the graph is concave down on that interval

    $f''(x)$ switches from $+$ to $-$ or vice versa means there is an inflection point at that x-value

    *to find the y-value of the max/min, plug into the *original equation*



To find Absolute Extrema…

1) Make a list of the x-values of all relative extrema and the endpoints of the given interval.
2) Calculate the y-value for each x-value.
3) The absolute max will be the point with the highest y-value. The absolute min will be the point with the lowest y-value.

Optimization Problems

1) Draw and label a picture.
2) Write two equations: the primary equation for what you need to maximize or minimize AND the secondary equation based on the facts given in the problem
3) Plug the secondary equation into the primary equation so that you have an equation in terms of one variable
4) Take the 1st derivative and analyze.
5) Answer the question that is asked.