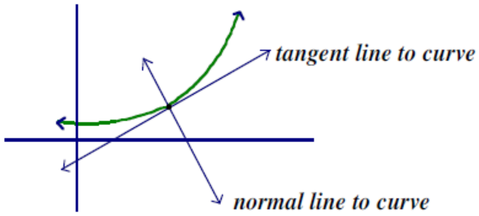
Derivatives

Derivative = slope of the tangent line to the curve

- 

A normal line is perpendicular to the tangent line.

Derivative Rules

Differentiation Rules	
Constant Rule	$\frac{d}{dx}[c]=0$
Power Rule	$\frac{d}{dx}x^n = nx^{n-1}$
Product Rule	$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Derivatives of basic functions.

$$\frac{d}{dx}[c] = 0 \quad (c, \text{ constant}) \qquad \frac{d}{dx}[x^n] = nx^{n-1}$$

$$\frac{d}{dx}[e^x] = e^x \qquad \frac{d}{dx}[a^x] = a^x \ln a$$

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x} \qquad \frac{d}{dx}[\log_a |x|] = \frac{1}{x \ln a}$$

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}[\csc^{-1} x] = -\frac{1}{x\sqrt{x^2-1}}$$

Higher Order Derivatives

First Derivative	$y',\quad f'(x),\quad F'(x),\quad \frac{dy}{dx},\quad \frac{d}{dx}[f(x)],\quad D_x[y]$
Second Derivative	$y'',\quad f''(x),\quad F''(x),\quad \frac{d^2y}{dx^2},\quad \frac{d^2}{dx^2}[f(x)],\quad D_x^2[y]$
Third Derivative	$y''',\quad f'''(x),\quad F'''(x),\quad \frac{d^3y}{dx^3},\quad \frac{d^3}{dx^3}[f(x)],\quad D_x^3[y]$
Fourth Derivative	$y^{(4)},\quad f^{(4)}(x),\quad F^{(4)}(x),\quad \frac{d^4y}{dx^4},\quad \frac{d^4}{dx^4}[f(x)],\quad D_x^4[y]$
n^{th} Derivative	$y^{(n)},\quad f^{(n)}(x),\quad F^{(n)}(x),\quad \frac{d^ny}{dx^n},\quad \frac{d^n}{dx^n}[f(x)],\quad D_x^n[y]$

Derivative of an Inverse

if $f^{-1}(x) = g(x)$

$$f(g(x)) = x$$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

Implicit Differentiation—use this process when the equation is not solved for y

- Differentiate both sides of the equation with respect to x.
- Apply the rules of differentiation. If an expression involves y, then include $\frac{dy}{dx}$.
- Isolate all the $\frac{dy}{dx}$ terms on one side of the equation.
- Factor out $\frac{dy}{dx}$.
- Solve for $\frac{dy}{dx}$ by dividing.

Limit Definition of a Derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiable $\Rightarrow$ Continuous

Differentiability Implies Continuity

If f is differentiable at x, then f is continuous at x.

Some things which destroy differentiability:

1. A discontinuity (a hole or break or asymptote)
2. A sharp corner (ex. $f(x) = |x|$ when $x = 0$)
3. A vertical tangent line (ex: $f(x) = \sqrt[3]{x}$ when $x = 0$)