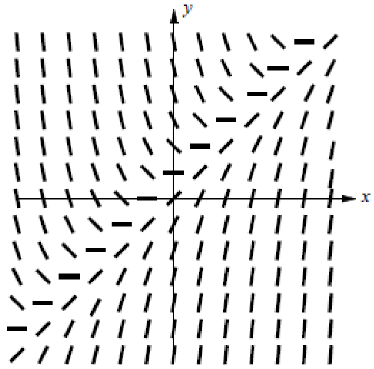
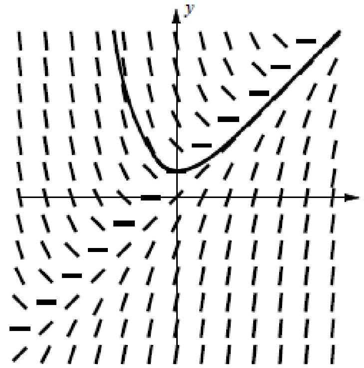
Differentials and Slope Fields

SLOPE FIELD

A first order differential equation of the form $y' = f(x, y)$ says that the slope of a solution curve at a point (x, y) on the curve is $f(x, y)$. If we draw short line segments with slope $f(x, y)$ at several points (x, y), the result is called a **slope field**.



Slope field for $y' = x - y + 1$



Particular solution for $y' = x - y + 1$ passing through (0,1)

DIFFERENTIAL EQUATION

When you solve a differential equation, you are given a derivative and need to find the original equation. We do this by using "separation of variables" which means we separate the *x*'s and *y*'s and take the integral.

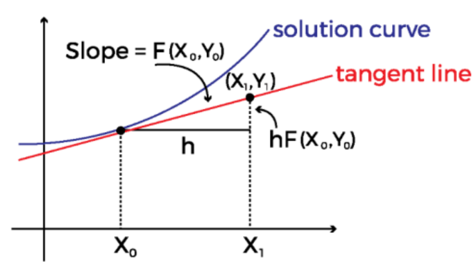
Steps:

1. Separate the "like variables" on opposite sides of the equation.
2. Integrate both sides.
3. If given, use the initial condition to determine the "C" value.
4. Solve for y.

EULER'S METHOD

Euler's Method is a numerical approach to approximate the particular solution of a differential equation with an initial condition.

$y - y_0 = m(x - x_0) \quad \Rightarrow \quad y = y_0 + \frac{dy}{dx}(\Delta x)$ where $\Delta x = step\ size$



LOGISTIC DIFFERENTIAL EQUATION

Logistic differential equation has form $\frac{dp}{dt} = kp\left(1 - \frac{p}{L}\right)$

p = population at time t
L = carrying capacity (max size of the population)
k = constant of proportionality

- The solution of a logistic differential equation has form $p = \frac{L}{1+be^{-kt}}$
- The population is growing the fastest when $p = \frac{L}{2}$, (half of the carrying capacity). This is also where the graph of p(t) has a point of inflection.
- $\lim_{t\to\infty} p(t) = L$
- $\lim_{t\to\infty} \frac{dp}{dt} = 0$
- If $0 < p < L$, then $1 - \frac{p}{L}$ is positive $\Rightarrow$ $\frac{dp}{dt} > 0$ $\Rightarrow$ the population increases
- If $p > L$, then $1 - \frac{p}{L}$ is negative $\Rightarrow$ $\frac{dp}{dt} < 0$ $\Rightarrow$ the population decreases