Integration

Basic Integration Rules

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
5. $\int e^u\,du = e^u + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \tan u\,du = -\ln|\cos u| + C$
9. $\int \cot u\,du = \ln|\sin u| + C$
10. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u\,du = \tan u + C$
13. $\int \csc^2 u\,du = -\cot u + C$
14. $\int \sec u \tan u\,du = \sec u + C$
15. $\int \csc u \cot u\,du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

Fundamental Theorem of Calculus

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

Second Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Properties of Definite Integrals

Definition

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Constant Multiple

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

Sum and Difference

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

Additivity

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

Riemann Sums

Let f be a continuous function defined on the closed interval $[a,b]$, and let Δ be a partition of $[a,b]$ given by

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

where Δx_i is the width of the i th interval. If c_i is any point in the i th interval, then the sum

$$\sum_{i=1}^{n} f(c_i)\Delta x_i = f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \cdots + f(c_i)\Delta x_i + \cdots + f(c_n)\Delta x_n$$

is called a **Riemann sum** for f on the interval $[a,b]$.

If every subinterval is of equal width, the partition is **regular** and $\Delta x = \dfrac{b-a}{n}$.

Then the Riemann sum can be written

$$\sum_{i=1}^{n} f(c_i)\Delta x = \Delta x\left[f(c_1) + f(c_2) + \cdots + f(c_i) + \cdots + f(c_n)\right]$$

where $c_i = a + i(\Delta x)$.

Trapezoidal Approximation

Let f be continuous on $[a,b]$. The **trapezoidal rule** for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right],$$

where $x_0 = a,\ x_1 = a + \Delta x,\ x_2 = a + 2\Delta x,\ \ldots,\ x_{n-1} = a + (n-1)\Delta x,\ x_n = b$, and $\Delta x = \dfrac{b-a}{n}$.