Limits and Continuity

Limits Approaching Infinity—check the powers of the numerator and denominator

- If the denominator has a larger power, the limit = 0.
- If the numerator has a larger power, the limit is ∞ or $-\infty$.
- If the powers are the same, the limit = $\dfrac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$

Limits Approaching a Real Number—check both the left- and right-limits

- If the left and right hand limits agree, then the limit is that value.
- If the left and right hand limits disagree, then the limit does not exist at that point.

$\lim_{x\to a^+} f(x) = \text{right hand limit}$ $\qquad$ $\lim_{x\to a^-} f(x) = \text{left hand limit}$

Special Trig Limits

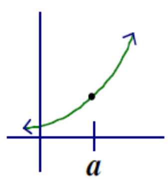
$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ $\qquad$ $\lim_{x\to 0} \dfrac{1-\cos x}{x} = 0$

L'Hopital's Rule—a method used to evaluate a limit for an indeterminate form $\dfrac{0}{0}$ or $\pm\dfrac{\infty}{\infty}$

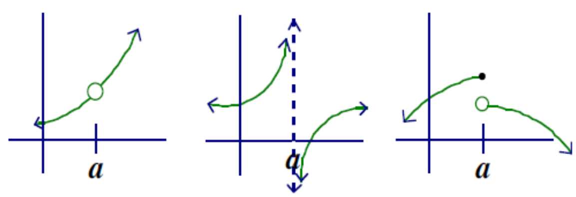
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Continuous Function—has no breaks

A discontinuous function could have a hole, asymptote, or a jump.



Continuous at a



Discontinuous at a

To check for **continuity at a point** "a", you must:

1) calculate the left- and right-hand limits approaching the value "a"
2) calculate the function value at "a"
3) state whether or not all 3 values are equal: equal $\Rightarrow$ continuous at "a" , not equal $\Rightarrow$ discontinuous at "a"

If $f(a) = \lim_{x\to -a^-} f(x) = \lim_{x\to -a^+} f(x)$ then the function is **continuous at *a***.

Limit Definition of a Derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$