Position, Velocity, Acceleration (PVA)/Motion

Let $s(t)$ be the **position function** for an object moving along a straight line.

Velocity is the derivative of position with respect to time.

$$v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = s'(t) .$$

Speed is the absolute value of velocity.

$$\text{speed} = |v(t)|$$

Acceleration is the derivative of velocity with respect to time.

$$a(t) = v'(t) = s''(t)$$

If $v > 0$, then the particle is **moving to the right**.
If $v < 0$, then the particle is **moving to the left**.

If $a > 0$, then v **is increasing**.
If $a < 0$, then v **is decreasing**.

If a and v have the same sign, the particle's **speed is increasing**.
If a and v have the opposite signs, the particle's **speed is decreasing**.

Note: For movement on a horizontal line, $x(t)$ is used to represent the position function; rightward movement is considered to be in the positive direction.
For movement on a vertical line, $y(t)$ is used to represent the position function; upward movement is considered to be in the positive direction.

If a particle moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$, so

$$s(b) - s(a) = \int_a^b v(t)\, dt$$

is the change of the position, or displacement, of the particle during the time period from $t = a$ to $t = b$.

The average velocity and the average acceleration over the time interval from $t = a$ to $t = b$ is

$$\textbf{Average velocity} = \frac{\textbf{displacement}}{\textbf{time}} = \frac{s(b) - s(a)}{b - a} = \frac{1}{b-a} \int_a^b v(t)\, dt$$

$$\textbf{Average acceleration} = \frac{v(b) - v(a)}{b - a} = \frac{1}{b-a} \int_a^b a(t)\, dt .$$

To find the total distance traveled we have to consider when the particle moves to the right, $v(t) \geq 0$, and when the particle moves to the left, $v(t) \leq 0$.
In both cases, the distance is computed by integrating $|v(t)|$, the speed of the particle.
Therefore

$$\textbf{Total distance traveled} = \int_a^b |v(t)|\, dt$$

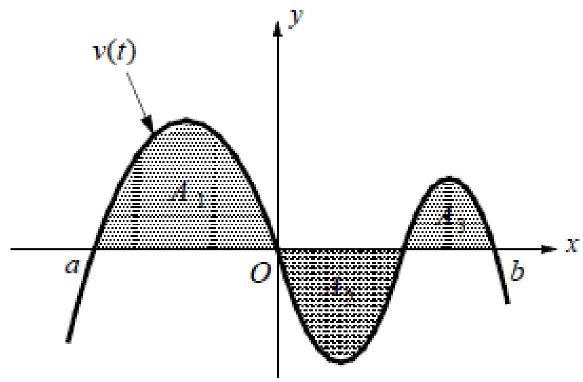
$$\textbf{Average speed} = \frac{\textbf{distance traveled}}{\textbf{time}} = \frac{1}{b-a} \int_a^b |v(t)|\, dt .$$

The acceleration of the object is $a(t) = v'(t)$, so

$$v(b) - v(a) = \int_a^b a(t)\, dt$$

is the change in velocity from $t = a$ to $t = b$.

The figure at the right shows how both displacement and distance traveled can be interpreted in terms of areas under a velocity curve.



displacement

$$= \int_a^b v(t)\, dt = A_1 - A_2 + A_3$$

total distance traveled

$$= \int_a^b |v(t)|\, dt = A_1 + A_2 + A_3$$