Pre- Calculus

REVIEW FOR MIDTERM

Chapter 1 – Functions and their properties:

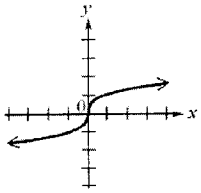
1) State the domain of each of the functions below:

a) $f(x) = \sqrt{3x-4}$ b) $f(x) = \dfrac{\sqrt{7-x}}{x-4}$ c) $f(x) = \ln(3x-7)$ d) $f(x) = \dfrac{\sqrt{3-x}}{2x^2+x-}$

$\left[\frac{4}{3}, \infty\right)$ $(-\infty, 4) \cup (4, 7]$ $\left(\frac{7}{3}, \infty\right)$ $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, 1\right) \cup (1, 3$

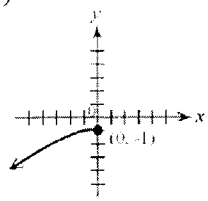
2) State the interval over which each of the following are increasing/decreasing:

a)



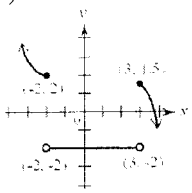
incr $(-\infty, \infty)$

b)



incr $(-\infty, 0]$

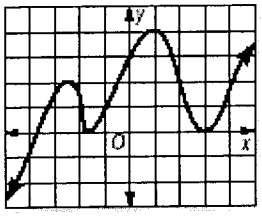
c)



decr $(-\infty, -2) \cup (3, \infty)$

3) Which of the following are functions? A & D

a)



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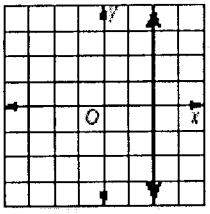
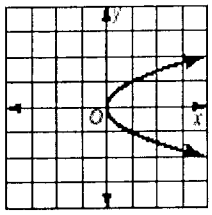
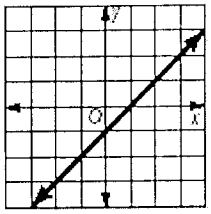


c)



d)



4) Determine whether each of the following are even, odd, or neither:

a) $f(x) = \dfrac{3x}{x^2-4}$ b) $y = 5|x| - 7x^4 + 11$ c) $f(x) = 5x^7 - 4x^3 - x$ d) $y = \sqrt{2x-9}$

ODD EVEN ODD NEITHER

5) Determine the functions implicitly defined below:

a) $x^2 + y^2 = 9$

$y = \pm\sqrt{-x^2+9}$

b) $y^2 - x = 2$

$y = \pm\sqrt{x+2}$

c) $9x^2 + 16y^2 = 100 - 24xy$

$y = -\frac{3}{4}x \pm \frac{5}{2}$

6) Use the given functions to find the compositions and their domains

$f(x) = x^2 - 3x$ $\quad g(x) = \dfrac{3}{x+1}$ $\quad h(x) = 2\sqrt{1-x}$ $\quad k(x) = \sqrt{x+3}$

a) $(f \circ k)(x)$

$x+3-3\sqrt{x+3}$

$[-3, \infty)$

b) $(h \circ g)(x)$

$2\sqrt{\frac{x-2}{x+1}}$

$(-\infty, -1) \cup [2, \infty)$

c) $g(f(x))$

$\dfrac{3}{x^2-3x+1}$

$x \neq \frac{3 \pm \sqrt{5}}{2}$

d) $f(h(x))$

$4 - 4x - 6\sqrt{1-x}$

$(-\infty, 1]$

7) Each function below is a transformation of one of the 12 basic functions. Describe the transformations.

a) $f(x) = e^{-2x+8} - 7$

right 4
horiz. shrink $* \frac{1}{2}$
refl. over y-axis
down 7

b) $g(x) = \dfrac{1}{2 + 2e^{-x+5}}$

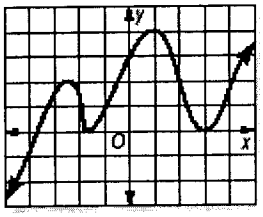
shift right 5
vert. shrink $* \frac{1}{2}$

c) $f(x) = 9\cos(2x) - 11$

horiz. shrink $* \frac{1}{2}$
vert. stretch $* 9$
down 11

8) Which of the following have inverses that are functions? B, C, & D

a)



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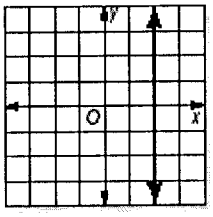
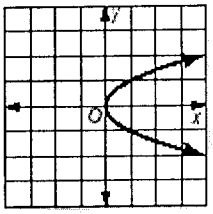
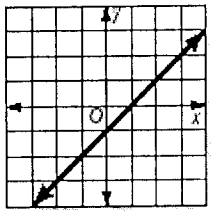


c)



d)



Chapter 2A – Polynomial & Power Functions:

1) Write the following quadratic functions in vertex form then state the vertex:

a) $f(x) = x^2 - 8x + 11$

$f(x) = (x-4)^2 - 5$

$V(4, -5)$

b) $y = 3x^2 + 6x + 5$

$y = 3(x+1)^2 + 2$

$V: (-1, 2)$

2) Find the zeros of the polynomial functions below:

a) $f(x) = x^3 - 3x^2 - 25x + 75$

$x = 5, -5, 3$

b) $f(x) = x^4 - 11x^2 + 28$

$x = \pm 7, \pm 2$

c) $f(x) = x^3 - 2x^2 - 5x + 6$

$x = 1, 3, -2$

3) Use long division or synthetic (you should look in your notes to determine which to use when) to find:

a) $\dfrac{3x^2 - 7x + 2}{x + 5}$

$3x - 22 + \dfrac{112}{x+5}$

b) $\dfrac{2x^4 - x^3 - 2}{2x^2 + x + 1}$

$x^2 - x + \dfrac{x-2}{2x^2+x+1}$

4) State the complex conjugate of each of the following and then find their product:

a) $7 - 3i$

$7 + 3i$

58

b) $-5 + 8i$

$-5 - 8i$

89

5) Find all the complex zeros of the following polynomial functions:

a) $y = x^3 - 2x^2 + 4x - 8$

$x = 2, \pm 2i$

b) $f(x) = x^2 - 6x + 25$

$x = 3 \pm 4i$

c) $y = 7x^2 + 42$

$x = \pm i\sqrt{6}$

6) Solve the following non-linear inequalities:

a) $(3x - 4)^2(x + 5)(x - 3) > 0$

$(-\infty, -5) \cup (3, \infty)$

b) $x^3 + 9x^2 + 11x \leq 21$

$(-\infty, -7] \cup [-3, 1]$

c) $\dfrac{x-8}{x^2-4} \geq \dfrac{-1}{x+2}$

$(-2, 2) \cup [5, \infty)$

7) Determine whether each of the following is a polynomial. If it is state the degree & leading coefficient (if not put "n/a"):

a. $f(x) = 5x^{3.2} + 4x - 3$

This is not (is/is not) a polynomial.

~~Degree = ___ & L.C. = ___~~

b. $f(x) = 7x^2 - 3x + 8$

This is (is/is not) a polynomial.

Degree = 2 & L.C. = 7

c. $f(x) = \dfrac{7x+5}{3x}$

This is not (is/is not) a polynomial.

~~Degree = ___ & L.C. = ___~~

d. $f(x) = \dfrac{7x+5}{3}$

This is (is/is not) a polynomial.

Degree = 1 & L.C. = $\frac{7}{3}$

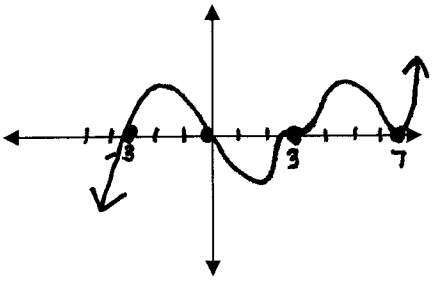
e. $f(x) = \sqrt[3]{18x^3 + 4x^2 + 10}$

This is not (is/is not) a polynomial.

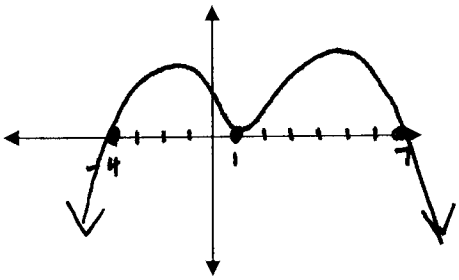
~~Degree = ___ & L.C. = ___~~

8) Sketch each of the following polynomials:

a) $y = x(x-3)^3(x+3)(x-7)^2$



b) $f(x) = (7-x)(x-1)^2(x+4)$



Chapter 2B – Rational Functions & Non-Linear Inequalities:

1) Simplify each of the following rational expressions. State any restricted values.

a) $\dfrac{x^2+6x+5}{x^2+3x-10}$

$\dfrac{x+1}{x-2}$

$x \neq -5, 2$

b) $\dfrac{20x^3+30x^2+45x}{40x^4-135x}$

$\dfrac{1}{2x-3}$

$x \neq 0, 3/2$

c) $\dfrac{\frac{5}{x-2}}{\frac{1}{x-2}+\frac{2}{x+1}}$

$\dfrac{5(x+1)}{3(x-1)}$

$x \neq 2, -1, 1$

2) Determine each of the x- & y-intercepts and state the left and right end behavior for each of the following:

a. $f(x) = x^4 - 5x^2 + 4$

y-int: (0,4)

Zero(s): 2, -2, 1, -1

LEB: ∞

REB: ∞

b. $f(x) = x^4 - 2x^3 + 8x - 16$

y-int: (0,-16)

Zero(s): 2, -2

LEB: ∞

REB: ∞

c. $f(x) = 81x^4 - 16$

y-int: (0,-16)

Zero(s): $\frac{2}{3}, -\frac{2}{3}$

LEB: ∞

REB: ∞

3) Write the equations of all of the asymptotes of the function:

a) $y=\dfrac{5x^3-4x^2+1}{x^3-8}$

V.A. $x=2$
H.A. $y=5$

b) $y=\dfrac{5x^2-20x^2+20}{x^3-8}$

V.A. $x=2$
H.A. $y=0$

c) $y=\dfrac{x-10}{x^2-9}$

V.A. $x=-3,3$
H.A. $y=0$

4) Solve each of the following rational equations

a) $\dfrac{x-2}{x+4}+\dfrac{x+1}{x+6}=\dfrac{11x+32}{x^2+10x+24}$

$x=5$

b) $\dfrac{y+3}{y+2}=1-\dfrac{y+1}{y+2}$

no soln

Chapter 3 – Logistic, Exponential, & Logarithmic Functions:

1) Write each of the following in exponential form:

a) $\log 7=x$

$10^x=7$

b) $\log_4 16=2$

$4^2=16$

c) $\log_u k=z$

$u^z=k$

2) Write each of the following in logarithmic form:

a) $7^3=343$

$\log_7 343=3$

b) $11^x=3$

$\log_{11} 3=x$

c) $t^4=11$

$\log_t 11=4$

3) Evaluate each of the following logarithms:

a) $\log_8 4=\dfrac{2}{3}$

b) $\log_9 27^{1/3}=\dfrac{1}{2}$

c) $\ln e^{4x}=4x$

d) $\log 0.001=-3$

4) Expand each of the following logarithms:

a) $\ln(7xy)$

$\ln 7+\ln x+\ln y$

b) $\ln\left(\dfrac{x(x+4)}{\sqrt{x^2+1}}\right)$

$\ln x+\ln(x+4)-\frac{1}{2}\ln(x^2+1)$

c) $\log\left(\dfrac{x^3y^6}{\sqrt{z}}\right)$

$3\log x+6\log y-\frac{1}{2}\log z$

5) Condense each of the following logarithms:

a) $\log_2 x-2\log_2 y+\dfrac{1}{3}\log_2 t$

$\log_2\left(\dfrac{x\cdot\sqrt[3]{t}}{y^2}\right)$

b) $4\ln a-\left(\ln b-\dfrac{2}{3}\ln c\right)$

$\ln\left(\dfrac{a^4\sqrt[3]{c^2}}{b}\right)$

c) $3(5\ln x+2\ln y)+2(\ln y-7\ln x)$

$\ln(xy^8)$

6) Solve each of the following logarithmic or exponential equations:

a) $\log_3(x^2+8) - \log_3 4 = 3$

$x = \pm 10$

b) $\ln(x+7) + \ln(x+3) = \ln 77$

$x = 4$

c) $\log_4(x^2-3) + \log_4 10 = 1$

$x = \pm\sqrt{\frac{17}{5}}$

d) $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

$x = -\frac{1}{6}$

e) $9^{-3x} \cdot 9^{x} = 27$

$x = -\frac{3}{4}$

f) $16^{2x-3} \cdot 4^{-2x} = 2^4$

$x = 4$