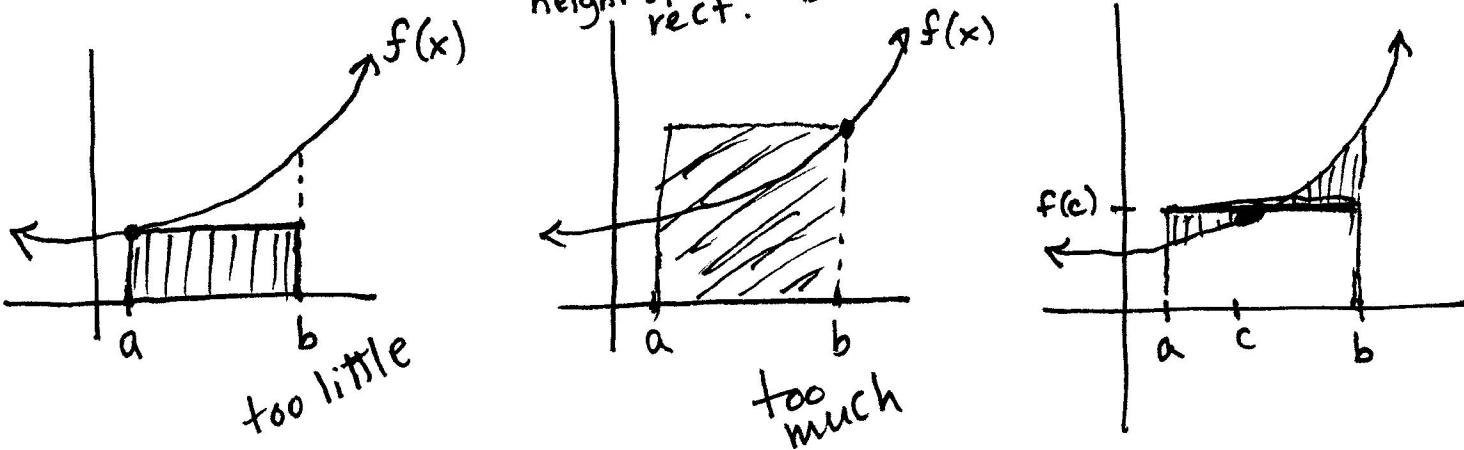


Mean Value Theorem for Integrals

11/6/18

If f is continuous on $[a, b]$, then there exists a # "c" in $[a, b]$ such that

$$\int_a^b f(x)dx = \underbrace{f(c)}_{\text{height of rect.}} \cdot \underbrace{(b-a)}_{\text{base length of rect.}}$$



$$\cancel{\text{if}} \quad f(c) = \frac{\int_a^b f(x)dx}{b-a} = \frac{1}{b-a} \int_a^b f(x)dx$$

average value

Ex 1 $f(x) = 3x^2 - 2x \quad [1, 4]$

Find the average value.

$$\begin{aligned} f(c) &= \frac{1}{4-1} \int_1^4 (3x^2 - 2x) dx = \frac{1}{3} \left[x^3 - x^2 + C \right]_1^4 \\ &= \frac{1}{3} [64 - 16 + C - (1 - 1 + C)] = \frac{1}{3} \cdot 48 = \boxed{16} \end{aligned}$$

EX2 Use the MVT for Integrals to find $f(c)$ and c for $f(x) = 2x^2$ over $[0, 3]$.

$$f(c) = \frac{1}{3-0} \int_0^3 2x^2 dx = \frac{1}{3} \left[\frac{2}{3}x^3 + C \right]_0^3$$

$$= \frac{1}{3} [18 + C - (0 + C)] = \boxed{6}$$

$$6 = 2x^2$$

$$3 = x^2$$

$$x = \pm\sqrt{3}$$

$$\boxed{c = \sqrt{3}}$$

Find: $\frac{d}{dx} \int_4^x (2t+3) dt$

$$\frac{d}{dx} \left[t^2 + 3t + C \Big|_4^x \right]$$

$$\frac{d}{dx} \left[x^2 + 3x + C - (16 + 12 + C) \right]$$

$$\frac{d}{dx} \left[x^2 + 3x - 28 \right]$$

$$2x + 3$$

$$\frac{d}{dx} \int_5^x (2t+3) dt$$

$$\frac{d}{dx} \left[t^2 + 3t + C \Big|_5^x \right]$$

$$\frac{d}{dx} \left[x^2 + 3x + C - (25 + 15 + C) \right]$$

$$\frac{d}{dx} \left[x^2 + 3x - 40 \right]$$

$$2x + 3$$

$$\frac{d}{dx} \int_{567}^x (2t+3) dt = 2x + 3$$

$$\frac{d}{dx} \int_4^{x^3} (2t+3)dt$$

$$\frac{d}{dx} \left[t^2 + 3t + C \Big|_4^{x^3} \right]$$

$$\frac{d}{dx} \left[x^6 + 3x^3 + C - (16 + 12 + C) \right]$$

$$\frac{d}{dx} \left[x^6 + 3x^3 - 28 \right]$$

$$6x^5 + 9x^2$$

$$3x^2(2x^3 + 3)$$

Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing " a ", then for every x in I :

$$\frac{d}{dx} \int_a^x f(t)dt = f(x)$$

if function of x , use the chain rule

Ex 3 Find the derivative:

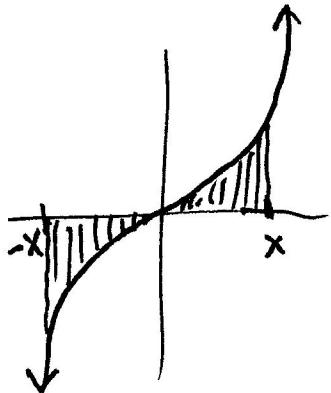
A. $F = \int_{\pi/2}^x \cos t dt$

$$\frac{d}{dx} \int_{\pi/2}^x \cos t dt = \cos x$$

B. $F = \int_{\pi/2}^{x^5} \cos t dt$

$$\frac{d}{dx} \int_{\pi/2}^{x^5} \cos t dt = 5x^4 \cos(x^5)$$

$$C.F = \int_{-x}^x t^3 dt$$



$$\frac{d}{dx} \int_{-x}^x t^3 dt$$

$$\frac{d}{dx} \left[\int_{-x}^0 t^3 dt + \int_0^x t^3 dt \right]$$

$$\frac{d}{dx} \left[- \int_0^{-x} t^3 dt + \int_0^x t^3 dt \right]$$

$$-1 \cdot (-x)^3 \circ -1 + x^3$$

$$-x^3 + x^3$$

O