

Antiderivatives

10/31/18

Given a derivative, find the original function

<u>$f'(x)$ derivative</u>	<u>original function $f(x)$</u>
$y' = 2x$	$y = x^2 + C$
$y' = x$	$y = \frac{1}{2}x^2 + C$
$y' = x^2$	$y = \frac{1}{3}x^3 + C$
$y' = 5x^4$	$y = x^5 + C$
$y' = \frac{1}{x^2} = x^{-2}$	$y = -x^{-1} + C$
$y' = \frac{1}{x^3} = x^{-3}$	$y = -\frac{1}{2}x^{-2} + C$
$y' = \cos x$	$y = \sin x + C$

differential equation - involves x, y & derivatives

variables
of
integration

indefinite integration

$$\frac{dy}{dx} = f'(x)$$

$$\int dy = \int f'(x) dx$$

$$y = f(x) + C$$

EX1 Find the general soln: $y' = 3x^2$

$$\frac{dy}{dx} = 3x^2$$
$$\int 1 dy = \int 3x^2 dx$$
$$y = x^3 + C$$

EX2 Evaluate:

A. $\int x^4 dx = \frac{1}{5}x^5 + C$

B. $\int (x^2 - 2x + 3) dx = \frac{1}{3}x^3 - x^2 + 3x + C$

C. $\int (\sqrt[4]{x^3} + 1) dx = \int (x^{3/4} + 1) dx = \frac{4}{7}x^{7/4} + x + C$

D. $\int \frac{x^2+1}{x^2} dx = \int \left(1 + \frac{1}{x^2}\right) dx = x + 1/x^{-1} + C$
 $x^{-2} \qquad \qquad \qquad = x - \frac{1}{x} + C$

E. $\int (2t^2 - 1)^2 dt = \int (4t^4 - 4t^2 + 1) dt$
 $= \frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C$

F. $\int (t^2 - \sin t) dt = \frac{1}{3}t^3 - (-\cos t) + C = \frac{1}{3}t^3 + \cos t + C$

G. $\int (\theta^2 + \sec^2 \theta) d\theta = \frac{1}{3}\theta^3 + \tan \theta + C$

$$\begin{aligned}
 \text{H. } \int \sec y (\tan y - \sec y) dy &= \int (\sec y \tan y - \sec^2 y) dy \\
 &= \sec y - \tan y + C
 \end{aligned}$$

$$\begin{aligned}
 \text{I. } \int \frac{\sin x}{1 - \sin^2 x} dx &= \int \frac{\sin x}{\cos^2 x} dx = \int \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \right) dx \\
 &= \int (\tan x \cdot \sec x) dx = \sec x + C
 \end{aligned}$$

$$\begin{aligned}
 \text{J. } \int \frac{\sqrt{x+3}}{x^2} dx &= \int \left(x^{-\frac{3}{2}} + 3x^{-2} \right) dx \\
 &\quad -2x^{-\frac{1}{2}} + 3x^{-1} + C \\
 &\quad \frac{-2}{\sqrt{x}} - \frac{3}{x} + C
 \end{aligned}$$

EX3 If $f'(s) = 6s - 8s^3$ and $\underbrace{f(2)}_{} = 3$, find $f(s)$.

$$\begin{aligned}
 \frac{dy}{ds} &= 6s - 8s^3 && (2, 3) \text{ initial condition} \\
 \int dy &= \int (6s - 8s^3) ds
 \end{aligned}$$

$$y = 3s^2 - 2s^4 + C \quad \leftarrow \text{general soln.}$$

$$3 = 3(2)^2 - 2(2)^4 + C$$

$$3 = 12 - 32 + C$$

$$23 = C$$

$$\boxed{f(s) = 3s^2 - 2s^4 + 23} \quad \swarrow \text{particular soln.}$$