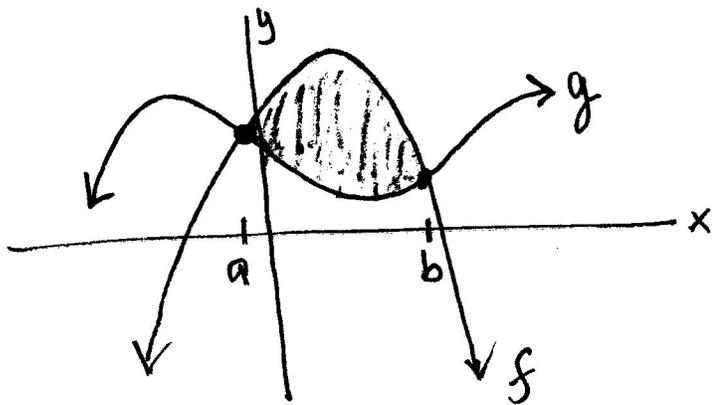


Area Between 2 Curves

12/17/18



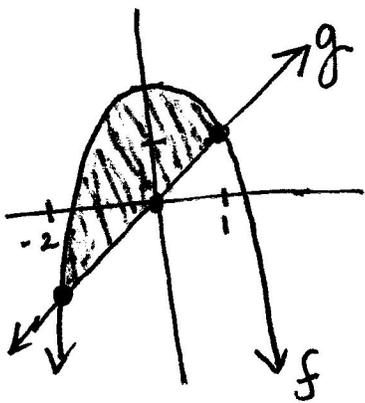
area of a shaded region

$$\int_a^b [f(x) - g(x)] dx$$

where $f(x) > g(x)$ for $[a, b]$

Find the area between the curves:

① $f(x) = 2 - x^2$ and $g(x) = x$



$$\begin{aligned} 2 - x^2 &= x \\ 0 &= x^2 + x - 2 \\ 0 &= (x + 2)(x - 1) \\ x &= -2, 1 \end{aligned}$$

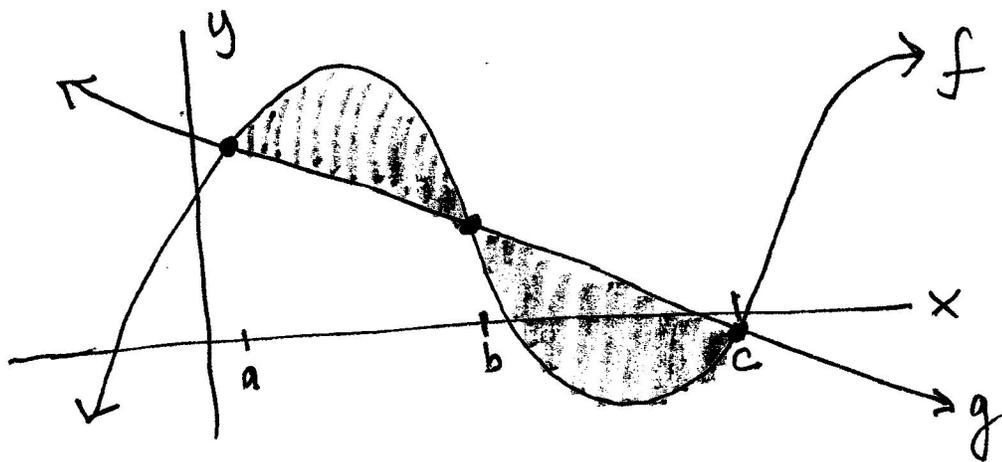
$$\int_{-2}^1 (2 - x^2 - x) dx$$

$$2x - \frac{1}{3}x^3 - \frac{1}{2}x^2 + C \Big|_{-2}^1$$

$$2 - \frac{1}{3} - \frac{1}{2} + C - \left(-4 + \frac{8}{3} - 2 + C \right)$$

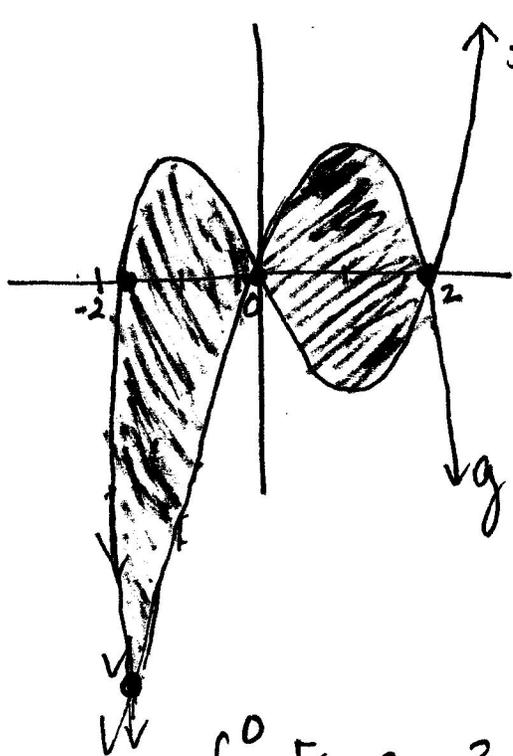
$$\boxed{\frac{9}{2}}$$

Sometimes the curves intersect in more than 2 points...



$$\int_a^b [f(x) - g(x)] dx + \int_b^c [g(x) - f(x)] dx$$

② $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$



$$3x^3 - x^2 - 10x = 0$$

$$x(3x^2 - x - 10) = 0$$

$$x(3x+5)(x-2) = 0$$

$$x = 0, -5/3, 2$$

$$-x(x-2) = 0$$

$$x = 0, 2$$

$$3x^3 - x^2 - 10x = -x^2 + 2x$$

$$3x^3 - 12x = 0$$

$$3x(x^2 - 4) = 0$$

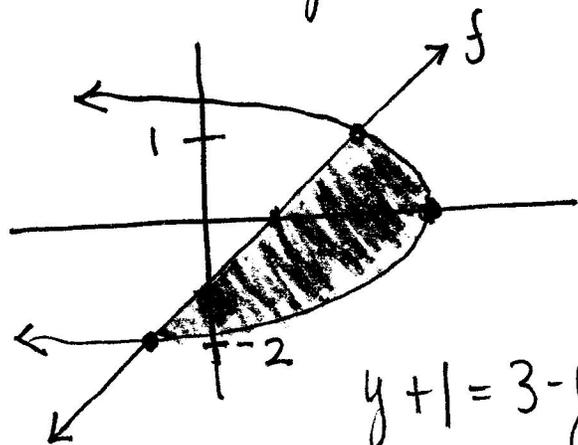
$$3x(x+2)(x-2) = 0$$

$$x = 0, -2, 2$$

$$\int_{-2}^0 [(3x^3 - x^2 - 10x) - (-x^2 + 2x)] dx + \int_0^2 [(-x^2 + 2x) - (3x^3 - x^2 - 10x)] dx$$

$$\int_{-2}^0 (3x^3 - 12x) dx + \int_0^2 (-3x^3 + 12x) dx = \boxed{24}$$

③ $f(y) = y+1$ and $g(y) = 3-y^2$
 $x = y+1$ $x = 3-y^2$
 $y = x-1$



$$y+1 = 3-y^2$$

$$y^2 + y - 2 = 0$$

$$(y+2)(y-1) = 0$$

$$y = -2, 1$$

right - left

$$\int_{-2}^1 [3-y^2 - (y+1)] dy$$

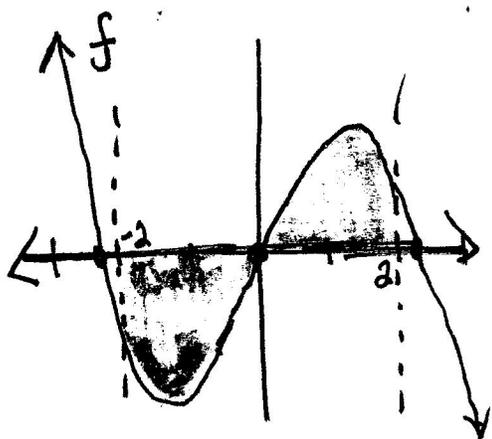
$$-y^2 - y + 2$$

$$-\frac{1}{3}y^3 - \frac{1}{2}y^2 + 2y + C \Big|_{-2}^1$$

$$-\frac{1}{3} - \frac{1}{2} + 2 + C - \left(+\frac{8}{3} - 2 - 4 + C \right)$$

$$\boxed{\frac{9}{2}}$$

④ $f(x) = -x^3 + 6x$ and the x-axis on $[-2, 2]$



$$-x(x^2 - 6) = 0$$

$$x = 0 \quad x^2 - 6 = 0$$

$$x = \pm\sqrt{6}$$

$$2 \int_0^2 [(-x^3 + 6x) - 0] dx = \boxed{16}$$