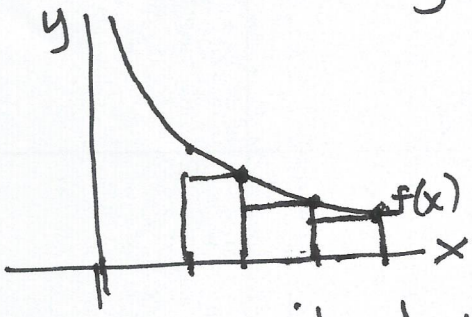
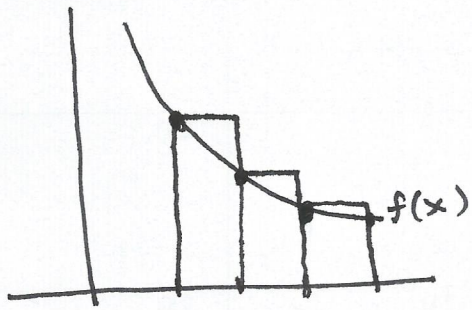
Area of a Region

inscribed rectangles – drawn below the curve



circumscribed rectangles – above the curve



"lower sum" – underestimate — inscribed rect

"upper sum" – overestimate — circumscribed rect

Riemann sums

LRAM – use left endpts.

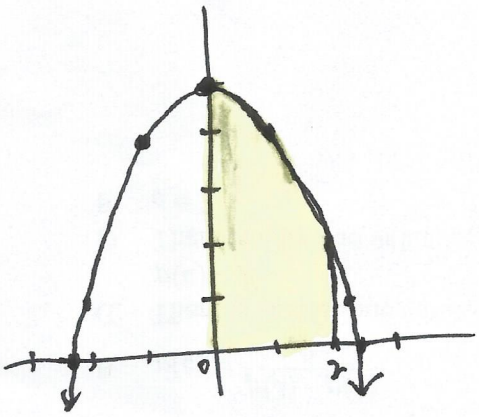
RRAM – use rt. endpts.

MRAM – use the midpoint of each subinterval

} to get the height of each rectangle

EX1 $f(x) = 5 - x^2$ for $[0, 2]$ $n=5$ subintervals

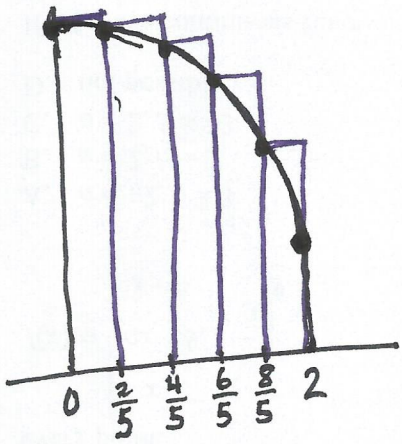
Find LRAM and RRAM.



$$\text{subinterval width} = \frac{\text{length of interval}}{\#\text{ subintervals}}$$

$$\frac{2-0}{5} = \frac{2}{5}$$

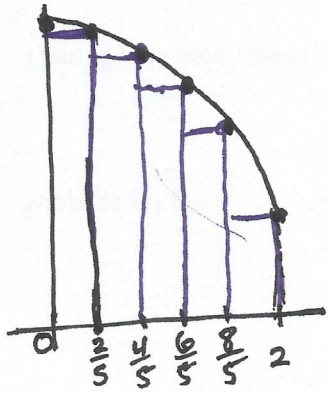
LRAM – overestimate



x	height $f(x)$
0	5
$\frac{2}{5}$	$\frac{121}{25}$
$\frac{4}{5}$	$\frac{109}{25}$
$\frac{6}{5}$	$\frac{89}{25}$
$\frac{8}{5}$	$\frac{61}{25}$

$$\frac{2}{5}\left(5 + \frac{121}{25} + \frac{109}{25} + \frac{89}{25} + \frac{61}{25}\right)$$
$$= 8.08$$

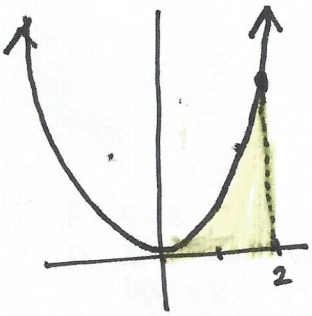
RRAM – underestimate



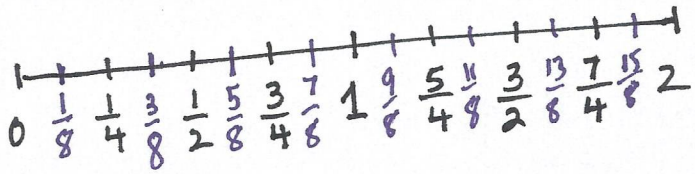
$$\frac{2}{5}\left(\frac{121}{25} + \frac{109}{25} + \frac{89}{25} + \frac{61}{25} + 1\right) = 6.48$$

EX2 $f(x) = x^2$, $n = 8$ subintervals, $[0, 2]$

Find LRAM, RRAM, & MRAM.



width subint. $= \dfrac{2-0}{8} = \dfrac{1}{4}$



underestimate

$$\text{LRAM} = \frac{1}{4}\left(0 + \frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1 + \frac{25}{16} + \frac{9}{4} + \frac{49}{16}\right) = 2.1875$$

$$\text{RRAM} = \frac{1}{4}\left(\frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1 + \frac{25}{16} + \frac{9}{4} + \frac{49}{16} + 4\right) = 3.1875$$

overestimate

$$\text{MRAM} = \frac{1}{4}\left(\frac{1}{64} + \frac{9}{64} + \frac{25}{64} + \frac{49}{64} + \frac{81}{64} + \frac{121}{64} + \frac{169}{64} + \frac{225}{64}\right) = 2.65625$$

EX3

	2	1	4	2
x	0	2	3	7	9
$f(x)$	3	6	7	6	8

Estimate the area using 4 subintervals with LRAM & RRAM.

$$\text{LRAM} = 2\cdot 3 + 1\cdot 6 + 4\cdot 7 + 2\cdot 6 = 52$$

$$\text{RRAM} = 2\cdot 6 + 1\cdot 7 + 4\cdot 6 + 2\cdot 8 = 59$$