Precalculus

COMPLEX NUMBERS REVIEW (2.5)

RECALL – The *imaginary number* is represented by the letter i

$i = \sqrt{-1}$

Now keeping in mind what we are allowed to do with equations.
What if we square both sides of that?

$i^2 = (\sqrt{-1})^2$

So that means that $i^2 = -1$ which is pretty interesting, because -1 is a REAL number!

RECALL - *Complex numbers*: These are numbers that have 2 parts to them, a real part, and an imaginary part.

The standard form of a complex number is $a \pm bi$. Where a = real & bi = imaginary

**********NOTE: complex numbers should ALWAYS be written in their standard form*********

OPERATIONS WITH COMPLEX NUMBERS

Essentially, when you are working with complex numbers, just follow all the rules of ALGEBRA you have learned up to this point, treating the i as if it were a variable (like x). Then, when you have finished your operation, and you are simplifying, replace any i^2 with -1 & finish simplifying.

***REMEMBER – you have to write your answer in the standard form, so real terms first then imaginary.

Practicing Operations with Complex Numbers:

Ex1) $(7 - 3i) + (4 + 5i) = 11 + 2i$

Ex2) $(5 + 2i) - (2 - i) = 3 + 3i$

Ex3) $(2 + i)(4 - i) = 8 - 2i + 4i - i^2 \rightarrow 8 + 2i - (-1) \rightarrow 9 + 2i$

Ex4) Given that $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ find the value of z^2 ~~and z^3~~

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$\frac{1}{4} + \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i + \frac{3}{4}\underbrace{i^2}_{(-1)} = \frac{1}{4} + \frac{\sqrt{3}}{2}i - \frac{3}{4}$$

$z^2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

~~$z^3 =$~~

Ok review is almost over… just ONE more thing…do you remember "***complex conjugates***?"
In short, every complex number has a complex conjugate that is ALMOST identical because both the real & imaginary are the same. However, the sign between them changes.

Ex5) Find the complex conjugate of each of the following: $a + bi$, $a - bi$

a) $4 - 2i$

$4 + 2i$

b) $-17 + 47i$

$-17 - 47i$

QUESTION: Why we need conjugates?

➔ **ANSWER:** We need them for division because all of our complex numbers have to be written in standard form. So when we need to get rid of a complex number in the denominator, we multiply (TOP & BOTTOM) by its complex conjugate, simplify, and write in std form.

Ex6) $\frac{2}{3-i}$

$$\frac{2(3+i)}{(3-i)(3+i)} = \frac{6+2i}{9+\cancel{3i}-\cancel{3i}-\underbrace{i^2}_{(-1)}} = \frac{6+2i}{10} = \frac{\cancel{2}(3+i)}{\cancel{10}\,5} = \boxed{\frac{3+i}{5}}$$

Ex7) $\frac{5+i}{2-3i}$

$$\frac{(5+i)(2+3i)}{(2-3i)(2+3i)} = \frac{10+15i+2i+3i^2}{4+\cancel{6i}-\cancel{6i}-9\underbrace{i^2}_{(-1)}} = \frac{10+17i-3}{4+9} = \boxed{\frac{7+17i}{13}}$$

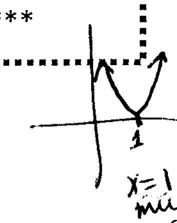
Precalculus

COMPLEX ZEROS & THE FUNDAMENTAL THEOREM OF ALGEBRA (2.6)

THEOREM: *Fundamental Theorem of Algebra* – A polynomial of degree n >0 has exactly n complex zeros.

NOTE: Complex includes: REAL, IMAGINARY, & COMPLEX*

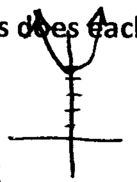
ALSO → the theorem doesn't use the word "distinct" which means they don't all have to be different. In other words we are allowed to have repeated zeros and it will count each and every time



x=1 mult. of 2

Ex1) How many complex zeros does each of the following have?

A) $f(x)=x^2+5$

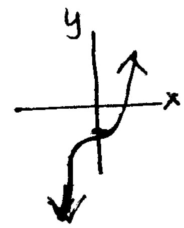


degree = 2

2 complex zeros

(both imaginary)

B) $f(x)=x^3-1$



deg = 3

3 complex zeros

(1 real, 2 imaginary)

THEOREM: *Polynomial Functions of Odd Degree* – have at least 1 real solution.

THEOREM: *Linear Factorization Theorem* – If polynomial degree $n>0$, then it has exactly n linear factors.

THEOREM: *Complex Conjugate Zeros* – If $a+bi$ is a solution, then $a-bi$ is a soln. too.

come in pairs

Ex2) Determine the zeros of the following polynomials...you are going to need the quadratic formula:

a) $f(x) = x^2 + x + 1$

$a=1 \quad b=1 \quad c=1$

$$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$$

$$=\frac{-1\pm\sqrt{1-4}}{2}$$

$$=\frac{-1\pm\sqrt{-3}}{2} \quad \sqrt{-1}\cdot\sqrt{3}$$

$$=\boxed{\frac{-1\pm i\sqrt{3}}{2}}$$

$$-\frac{1}{2}+\frac{i\sqrt{3}}{2} \qquad -\frac{1}{2}-\frac{i\sqrt{3}}{2}$$

b) $g(x) = x^3 - 27$

$$g(x)=(x-3)(x^2+3x+9)$$

$x-3=0$

$\boxed{x=3}$

$x^2+3x+9=0$

$a=1 \quad b=3 \quad c=9$

$$x=\frac{-3\pm\sqrt{(3)^2-4(1)(9)}}{2(1)}$$

$$=\frac{-3\pm\sqrt{9-36}}{2}$$

$$=\frac{-3\pm\sqrt{-27}}{2} \quad \sqrt{-1}\cdot\sqrt{27} \quad 9\cdot3$$

$$=\boxed{\frac{-3\pm 3i\sqrt{3}}{2}}$$

c) $h(x) = x^2 + 7$

$$x^2+7=0$$

$$x^2=-7$$

$$x=\pm\sqrt{-7}$$

$$x=\boxed{\pm i\sqrt{7}}$$

What if we were given one of the zeros, but it just happened to be complex…could you still do it?

Ex3) Given that $f(x) = 4x^4 + 17x^2 + 14x + 65$ and given that one zero of f(x) is $1 - 2i$, find the remaining zeros of f(x) and write it in its linear factorization.

deg = 4 ⇒ 4 complex zeros $(x-(1-2i))(x-(1+2i))(x-(-1+\frac{3}{2}i))(x-(-1-\frac{3}{2}i))$

if $1-2i$ is a zero, then $\boxed{1+2i}$ is also

x = # root
factor x − #

$x = 1-2i$

$[x-(1-2i)][x-(1+2i)]$

quad. factors
$(x^2-2x+5)(4x^2+8x+13)$

$x^2 - x(1+2i) - x(1-2i) + (1-2i)(1+2i)$

$x^2 - x - 2xi - x + 2xi + 1 + 2i - 2i \underbrace{-4i^2}_{+4}$

$x^2 - 2x + 5$

$$
\begin{array}{r}
4x^2 + 8x + 13 \\
x^2 - 2x + 5 \overline{\smash{)}\, 4x^4 + 0x^3 + 17x^2 + 14x + 65} \\
-(4x^4 - 8x^3 + 20x^2) \\
\hline
8x^3 - 3x^2 + 14x \\
-(8x^3 - 16x^2 + 40x) \\
\hline
13x^2 - 26x + 65 \\
-(13x^2 - 26x + 65) \\
\hline
0
\end{array}
$$

$4x^2 + 8x + 13 = 0$

$a = 4 \quad b = 8 \quad c = 13$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(4)(13)}}{2(4)} = \frac{-8 \pm \sqrt{64 - 208}}{8} = \frac{-8 \pm \sqrt{-144}}{8}$$

$$\frac{-8 \pm 12i}{8} = \frac{-2 \pm 3i}{2} = \boxed{-1 \pm \frac{3}{2}i}$$ 15