

## Unit #3 Notes – First & Second Derivative Tests

Match each term below with its correct definition.

### Important Terms:

- Critical Points
- Absolute (Global) Maximum
- Absolute (Global) Minimum
- Relative (Local) Maximum
- Relative (Local) Minimum
- Function is Increasing
- Function is Decreasing
- Function is Concave Up
- Function is Concave Down
- Points of Inflection

C  
I  
F  
H  
E  
B  
J  
G  
D  
A

### Definitions:

- A. function changes concavity/zeros of second derivative
- B. derivative is positive
- C. zeros of derivative or where derivative DNE
- D. second derivative is negative
- E. the point a graph changes from decreasing to increasing
- F. minimum of entire interval
- G. second derivative is positive
- H. the point a graph changes from increasing to decreasing
- I. maximum of entire interval
- J. derivative is negative

### ~~THE FIRST DERIVATIVE TEST~~ relative max/min for f incr./decr.

Suppose that  $c$  is a critical number of a continuous function  $f$ :

- If  $f'$  changes from positive to negative at  $c$ , then  $f$  has a local maximum at  $c$ .
- If  $f'$  changes sign from negative to positive at  $c$ , then  $f$  has local minimum at  $c$ .
- If  $f'$  does not change signs at  $c$ , then  $f$  has no local extreme value at  $c$ .
- At left endpoint  $a$  - If  $f' < 0$  for  $x > a$ , then  $f$  has a local maximum at  $a$ . If  $f' > 0$  for  $x > a$ , then  $f$  has a local minimum at  $a$ .
- At right endpoint  $b$  - If  $f' < 0$  for  $x < b$ , then  $f$  has a local minimum at  $b$ . If  $f' > 0$  for  $x < b$ , then  $f$  has a local maximum at  $b$ .

### ~~SECOND DERIVATIVE ANALYSIS~~ concavity for f POI

- If  $f'' > 0$  then the function is concave up
- If  $f'' < 0$  then the function is concave down

relative max/min for f  
incr/decr

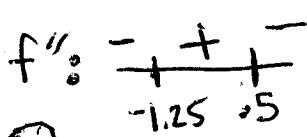
### ~~SECOND DERIVATIVE TEST~~ rel. max/min for f

Suppose that  $c$  is a critical number of a continuous function  $f$ :

- If  $f'(c) = 0$  and  $f''(c) < 0$ , then  $f$  has a local max at  $x = c$
- If  $f'(c) = 0$  and  $f''(c) > 0$ , then  $f$  has a local min at  $x = c$
- If  $f''(c) = 0$  or does not exist, the test fails. (When this happens, defer to the 1st derivative test.)

EX1) Given the graph of  $f'$ , identify the following for  $f$ :

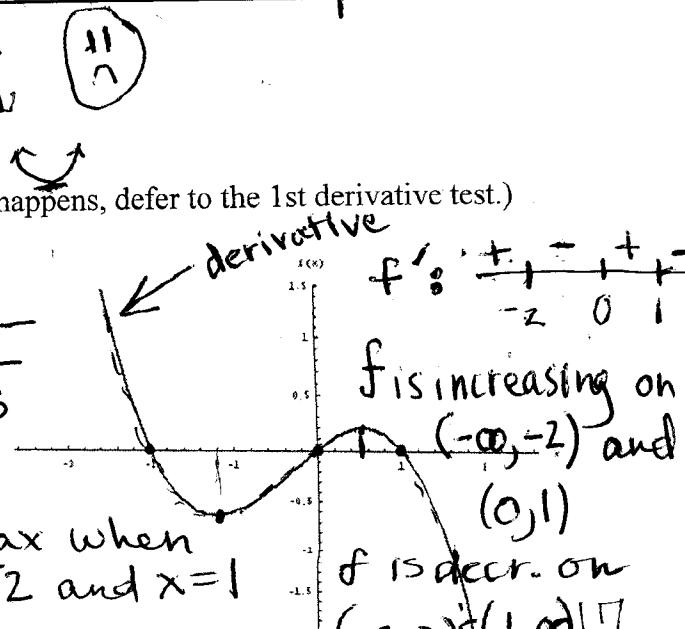
1. Maximums
2. Minimums
3. Intervals increasing/decreasing
4. Inflection points
5. Concavity



$f$  concave down  $(-\infty, -1.25)$   
and  $(0.5, \infty)$

$f$  concave up  $(-1.25, 0.5)$   
POI at  $x = -1.25$  and  $x = 0.5$

rel. max when  
 $x = -2$  and  $x = 1$   
rel. min when  
 $x = 0$



Ex2) Given the graph of  $f'$ , identify the following for  $f$ :

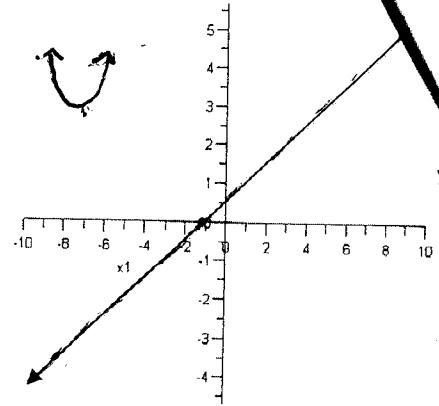
1. Maximums
2. Minimums
3. Intervals increasing/decreasing
4. Inflection points
5. Concavity

$$f': \begin{array}{c} - \\ + \\ - \end{array}$$

$f$  decr  $(-\infty, -1)$

$f''$ : pos. for every  $X$   $f$  incr  $(-1, \infty)$

$f$  concave up always      rel. min at  $x = -1$   
no poi      no rel. max



Ex3) Find all extrema on the given intervals:

a)  $f(x) = x^3 - 6x + 5$   $[-2, 3]$  abs max/min

$$\begin{aligned} f'(x) &= 3x^2 - 6 = 0 \\ 3(x^2 - 2) &= 0 \\ x^2 - 2 &= 0 \\ x &= \pm\sqrt{2} \end{aligned}$$

x	f(x)
$-\sqrt{2}$	10.657
$\sqrt{2}$	-6.657
-2	9
3	14

$(3, 14)$  abs. max

b)  $f(x) = 3x^{2/3}$   $[-1, 2]$

$$f'(x) = 3 \cdot \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$$

$f'(x)$  undefined at  $x = 0$ .

$$f': \begin{array}{c} - \\ + \\ 0 \end{array}$$

rel. min at  $x = 0$

rel. max at  $x = -\sqrt{2}$   
rel. min at  $x = \sqrt{2}$

$(\sqrt{2}, -6.657)$  abs. min

c)  $f(x) = \frac{1}{\sqrt{4-x^2}}$   $(-\infty, \infty)$

x	f(x)
0	0
-1	3
2	$3(2)^{2/3} = 4.762$

abs. max  $(2, 4.762)$

abs. min at  $(0, 0)$