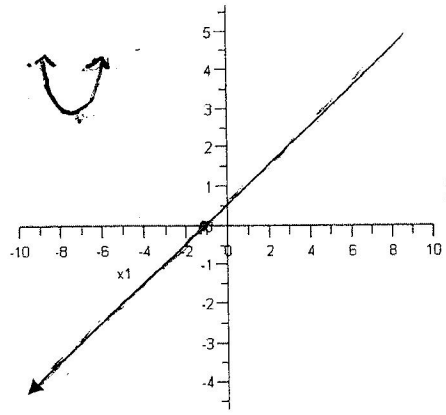
Ex2) Given the graph of f ', identify the following for f:

1. Maximums
2. Minimums
3. Intervals increasing/decreasing
4. Inflection points
5. Concavity

f': $-$ | $+$ (-1)



f decr $(-\infty, -1)$

f'': pos. for every x f incr $(-1, \infty)$

rel. min at $x = -1$

f concave up always no rel. max

no POI

Ex3) Find all extrema on the given intervals:

a) $f(x) = x^3 - 6x + 5$ $[-2, 3]$ abs max|min

$f'(x) = 3x^2 - 6 = 0$

$3(x^2 - 2) = 0$

$x^2 - 2 = 0$

$x = \pm\sqrt{2}$

x	$f(x)$
$-\sqrt{2}$	10.657
$\sqrt{2}$	$-.657$ ←
-2	9
3	14 ←

f': $+$ | $-$ ($-\sqrt{2}$) | $+$ ($\sqrt{2}$)

$(3, 14)$ abs. max

rel. max at $x = -\sqrt{2}$

rel. min at $x = \sqrt{2}$

$(\sqrt{2}, -.657)$ abs. min

b) $f(x) = 3x^{2/3}$ $[-1, 2]$

$f'(x) = 3 \cdot \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{\sqrt[3]{x}}$

$f'(x)$ undefined at $x = 0$

f': $-$ | $+$ (0)

rel. min at $x = 0$

x	$f(x)$
0	0
-1	3
2	$3(2)^{2/3} = 4.76$

abs. max $(2, 4.762)$

abs. min at $(0, 0)$

c) $f(x) = \frac{1}{\sqrt{4 - x^2}}$ $(-\infty, \infty)$ $\longrightarrow (4 - x^2)^{-1/2}$

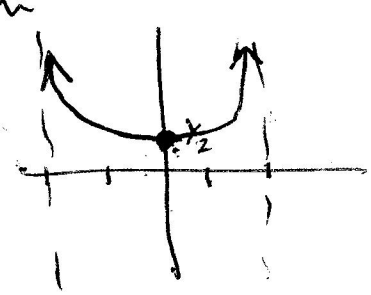
domain: $(-2, 2)$

$f'(x) = -\frac{1}{2}(4 - x^2)^{-3/2}(-2x) = \frac{x}{(4 - x^2)^{3/2}}$

$f'(x) = 0$ when $x = 0$

$f'(x)$ undefined when $x = \pm 2$

$f'(x)$: $-$ | $+$ (-2, 0, 2)



rel min at $x = 0$

no rel max

x	$f(x)$
0	$\frac{1}{2}$

$(0, \frac{1}{2})$ abs. min

no abs max

Ex4) Find all the intervals where $f(x) = 4x^3 - 3x^2 - 18x + 6$ is increasing & all intervals where f is decreasing.

$f'(x) = 12x^2 - 6x - 18 = 0$

$6(2x^2 - x - 3) = 0$

$6(2x-3)(x+1) = 0$

$2x - 3 = 0 \quad x + 1 = 0$

$x = 3/2 \quad x = -1$

f': $+ \quad - \quad +$; -1, $3/2$

f incr $(-\infty, -1)$ and $(\frac{3}{2}, \infty)$

f decr $(-1, 3/2)$

Ex5) Find the relative extrema of $y = \sin(x) - 2\cos(x)$ in the interval $[0, 2\pi]$. Justify your answer.

$y' = \cos x - 2(-\sin x)$

$0 = \cos x + 2\sin x$

$x = 2.678$ & $x = 5.820$

f': $[+ \quad - \quad +]$; 0, 2.678, 5.820, 2π

at $x = 2.678$ rel. max

at $x = 5.820$ rel. min

Ex6) Find the points of inflection for $g(x) = 3x^4 - 8x^3 + 6x^2$. Justify your answer.

$g'(x) = 12x^3 - 24x^2 + 12x$

$g''(x) = 36x^2 - 48x + 12 = 0$

$12(3x^2 - 4x + 1) = 0$

$12(3x - 1)(x - 1) = 0$

$x = 1/3 \quad x = 1$

f'': $+ \quad - \quad +$; $1/3$, 1

POI at $x = \frac{1}{3}$ & $x = 1$

$(\frac{1}{3}, \frac{11}{27})$

$(1, 1)$