recalculus Name: _____

Notes (4.4) --- Graphing Sinusoids

Definition: A function is a sinusoids if it can be written in the form → $f(x) = a \cdot \sin(bx + c) + d$

(~~were~~ where a, b, c, and d are constants and neither a ~~not~~ nor b is 0) **OR** $f(x) = a \cdot \cos(bx + c) + d$

Definition: The amplitude of a sinusoid of the form form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $|a|$. Graphically, the amplitude is $\frac{1}{2}$ of the total height.

Definition: The period of a sinusoid of the form form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $\frac{2\pi}{|b|}$. Graphically, the period is the length of one full cycle.

Definition: The frequency of a sinusoid of the form form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $\frac{|b|}{2\pi}$. Graphically, the frequency is the number of complete cycles in 2π radians

Definition: The phase shift of a sinusoid of the form form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $\frac{c}{b}$. Graphically, the phase shift is the transition to right or left

Ex1) Find the amplitude of each of the following sinusoids & then use the language of transformations to describe how the graphs of b and c are related to a.

a) $f(x) = \cos x$

amp = $|1| = 1$

b) $y = \frac{1}{2}\cos x$

amp = $\left|\frac{1}{2}\right| = \frac{1}{2}$

vert. shrink * $\frac{1}{2}$

c) $y = -3\cos x$

amp = $|-3| = 3$

vert. stretch * 3

reflect over x-axis

Ex2) Find the period of each of the following sinusoids & then use the language of transformations to describe how the graphs of b and c are related to a.

a) $f(x) = \sin x$

pd = $\frac{2\pi}{|1|} = 2\pi$

b) $y = 3\sin(-2x)$

pd = $\frac{2\pi}{|-2|} = \frac{2\pi}{2} = \pi$

horiz. shrink * $\frac{1}{2}$

refl. over y-axis

vert. stretch * 3

c) $y = -2\sin\left(\frac{x}{3}\right)$

pd = $\frac{2\pi}{\left|\frac{1}{3}\right|} = 2\pi \cdot \frac{3}{1} = 6\pi$

horiz. stretch * 3

vert. stretch * 2

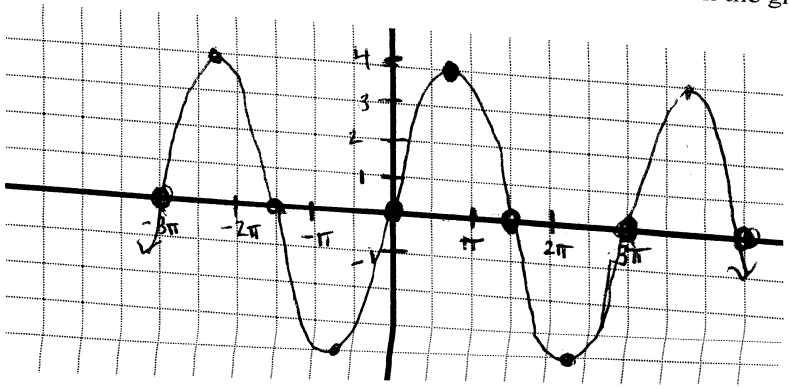
refl. over x-axis

Ex3) Find the frequency of the function $f(x) = 4\sin\left(\frac{2x}{3}\right)$ and interpret its meaning graphically. Then sketch the graph ~~in the window $[-2\pi, 2\pi]$ by $[-4, 4]$~~

freq = $\frac{|b|}{2\pi} = \frac{\left|\frac{2}{3}\right|}{2\pi} = \frac{2}{3} \cdot \frac{1}{2\pi} = \frac{1}{3\pi}$

amp = $|a| = |4| = 4$

per = $\frac{2\pi}{|b|} = 3\pi$

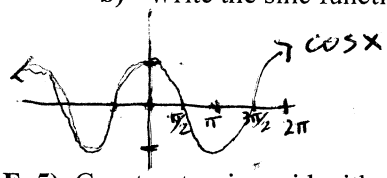


horiz. stretch * $\frac{3}{2}$

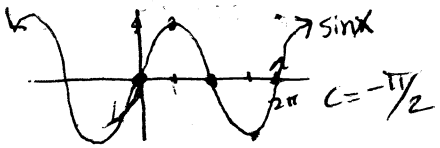
vert. stretch * 4

Ex4) **a)** Write the cosine function as a phase shift of the sine function. → $\cos(x) =$ $\sin\left(x + \frac{\pi}{2}\right)$

b) Write the sine function as a phase shift of the cosine function. → $\sin(x) =$ $\cos\left(x - \frac{\pi}{2}\right)$



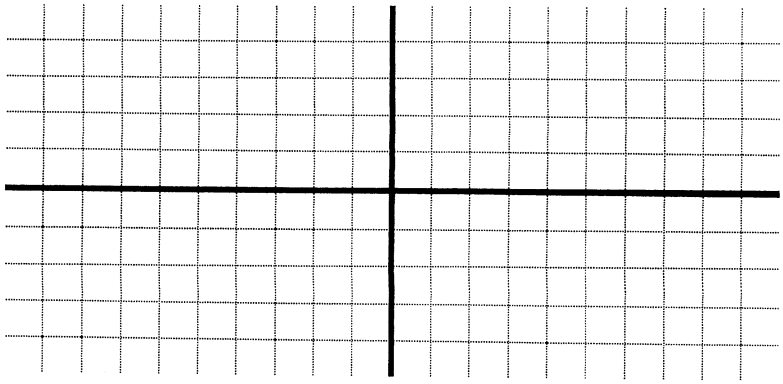
→ $c = \frac{\pi}{2}$



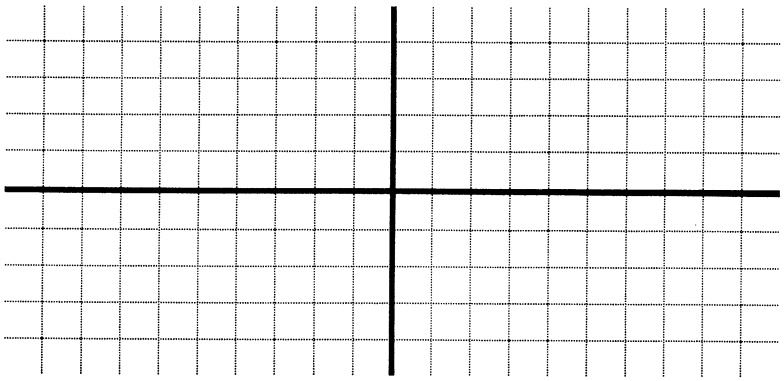
$c = -\pi/2$

Ex5) Construct a sinusoid with a period of $\frac{\pi}{5}$, amplitude 6, passing through the point (2, 0)

$f(x) =$ _____



Ex6) Construct a sinusoid that rises from a minimum value at (0, 5) to a maximum value of (32, 25)



Graphs of Sinusoids

The graphs of $y = a \sin(b(x - h)) + k$ and $y = a \cos(b(x - h)) + k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics:

amplitude $= |a|$;

period $= \frac{2\pi}{|b|}$;

frequency $= \frac{|b|}{2\pi}$.

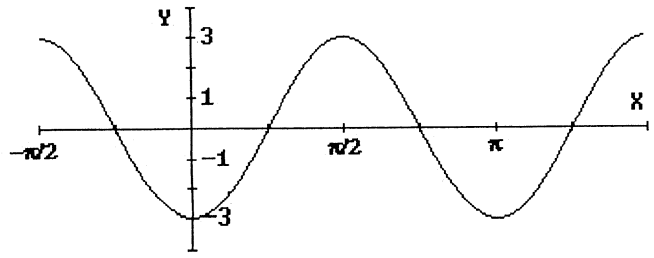
When compared to the graphs of $y = a \sin bx$ and $y = a \cos bx$, respectively, they also have the following characteristics:

a phase shift of h;

a vertical translation of k.

Graphing Worksheet

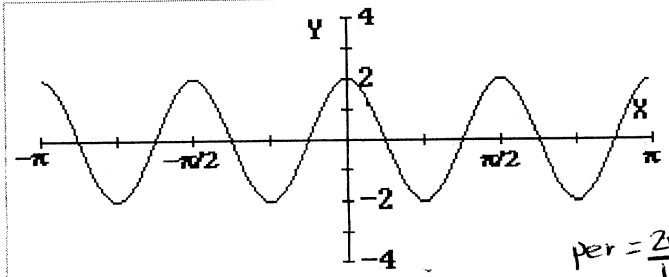
Find the amplitude, period and a sine ~~or~~ cosine function for each of the following graphs.
Your function should be of the form y = A sin(Bx) or y = A cos(Bx).



1. Amplitude _____ Period _____

Sine Function _____

Cosine Function _____

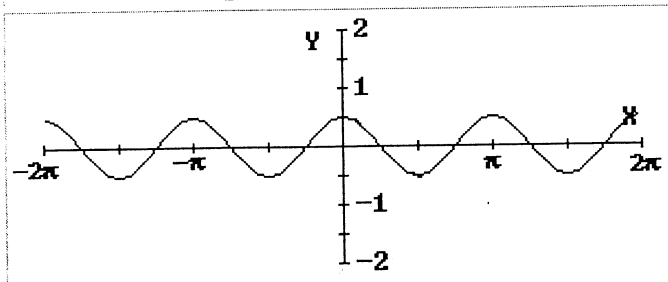


per = $\frac{2\pi}{|B|}$

2. Amplitude $\frac{1}{2}(4) = 2$ Period π $\frac{\pi}{2} = \frac{2\pi}{B}$

~~Sine Function~~ _____ $\pi B = 4\pi$

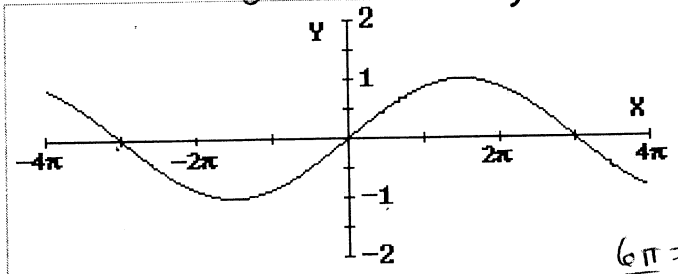
Cosine Function $y = 2\cos(4x)$ $B = \frac{4\pi}{\pi} = 4$



3. Amplitude _____ Period _____

Sine Function _____

Cosine Function _____

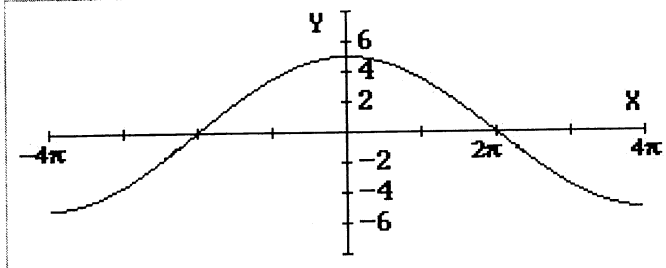


$\frac{6\pi}{1} = \frac{2\pi}{|B|}$

4. Amplitude $\frac{1}{2}(2) = 1$ Period 6π $6\pi B = 2\pi$

Sine Function $y = 1\sin(\frac{1}{3}x)$ $B = \frac{2\pi}{6\pi} = \frac{1}{3}$

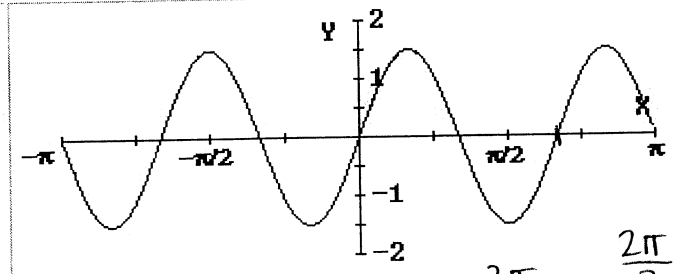
~~Cosine Function~~ _____



5. Amplitude _____ Period _____

Sine Function _____

Cosine Function _____



$\frac{2\pi}{3} = \frac{2\pi}{B}$

6. Amplitude $\frac{1}{2}(3) = \frac{3}{2}$ Period $\frac{2\pi}{3}$ $B = 3$

Sine Function $y = \frac{3}{2}\sin(3x)$

~~Cosine Function~~ _____