Notes--Group Ranking Methods

Example 1 The Math Appreciation Society is electing its president. The candidates are Alisha (A), Boris (B), Carmen (C), and Dave (D). Each of the 37 members votes with a preference ballot. Who should be the winner? Why?

Step 1: Everyone Votes

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *A*	1st *B*	1st *A*	1st *C*	1st *B*	1st *C*	1st *A*	1st *B*	1st *C*	1st *A*	1st *C*	1st *D*	1st *A*	1st *A*	1st *C*	1st *A*	1st *C*	1st *D*
2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *B*	2nd *C*	2nd *B*	2nd *B*	2nd *B*	2nd *B*	2nd *B*	2nd *C*
3rd *C*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *B*	3rd *C*	3rd *C*	3rd *D*	3rd *C*	3rd *D*	3rd *B*
4th *D*	4th *A*	4th *D*	4th *A*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *D*	4th *A*	4th *D*	4th *A*	4th *A*

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st *C*	1st *A*	1st *D*	1st *D*	1st *C*	1st *C*	1st *D*	1st *A*	1st *D*	1st *C*	1st *A*	1st *D*	1st *B*	1st *A*	1st *C*	1st *A*	1st *A*	1st *D*	1st *A*
2nd *B*	2nd *B*	2nd *C*	2nd *C*	2nd *B*	2nd *B*	2nd *C*	2nd *B*	2nd *C*	2nd *B*	2nd *B*	2nd *C*	2nd *D*	2nd *B*	2nd *D*	2nd *B*	2nd *B*	2nd *C*	2nd *B*
3rd *D*	3rd *C*	3rd *B*	3rd *B*	3rd *D*	3rd *D*	3rd *B*	3rd *C*	3rd *B*	3rd *D*	3rd *C*	3rd *B*	3rd *C*	3rd *C*	3rd *B*	3rd *C*	3rd *C*	3rd *B*	3rd *C*
4th *A*	4th *D*	4th *A*	4th *A*	4th *A*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *A*	4th *D*	4th *A*	4th *D*	4th *D*	4th *A*	4th *D*

Step 2: Collect Ballots that are exactly the same

# of ballots:	14	10	8	4	1
	Ballot	**Ballot**	**Ballot**	**Ballot**	**Ballot**
	1st *A*	1st *C*	1st *D*	1st *B*	1st *C*
	2nd *B*	2nd *B*	2nd *C*	2nd *D*	2nd *D*
	3rd *C*	3rd *D*	3rd *B*	3rd *C*	3rd *B*
	4th *D*	4th *A*	4th *A*	4th *A*	4th *A*

Step 3: Organize the data neatly in a chart called a "preference schedule"

Number of voters	14	10	8	4	1
1st choice	*A*	*C*	*D*	*B*	*C*
2nd choice	*B*	*B*	*C*	*D*	*D*
3rd choice	*C*	*D*	*B*	*C*	*B*
4th choice	*D*	*A*	*A*	*A*	*A*

1. How many first place votes did each candidate receive? A 14 C 11 B 4 D 8

2. Did anyone get a **majority** of the first place votes?

no

Majority =
MORE than half of the total first-place votes

$\frac{1}{2}(37) = 18.5$
majority ≥ 19

3. Who received the **plurality** of the first-place votes?

Alisha

Plurality =
the MOST first-place votes

Plurality Method: the Candidate with the most first-place votes wins

Advantages:

- Easy to determine
- Easy to understand
- Only need 1st choice (not necessarily a ranked ballot)

Most political elections in the USA are determined using the plurality method.

Sometimes using plurality has problems . . .

Number of voters	14	10	8	4	1
1st choice	*A*	*C*	*D*	*B*	*C*
2nd choice	*B*	*B*	*C*	*D*	*D*
3rd choice	*C*	*D*	*B*	*C*	*B*
4th choice	*D*	*A*	*A*	*A*	*A*

How many first place votes did our winner get?

14

Where is our winner on EVERYONE ELSE's BALLOT?

last

23 people

Example 2 The band has the choice to perform at 5 different bowl games: the Rose Bowl (R), the Hula Bowl (H), the Fiesta Bowl (F), the Orange Bowl (O), and the Sugar Bowl (S).

R 49
H 48
F 3
O 0
S 0

Who is the PLURALITY winner?

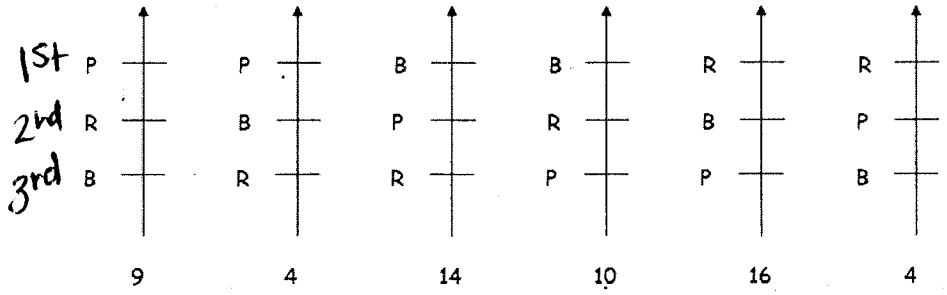
Rose

Is there a better choice? Why?

Hula — more preferred by everyone

Preference Schedule for the Band Election			
Number of voters	49	48	3
1st choice	*R*	*H*	*F*
2nd choice	*H*	*S*	*H*
3rd choice	*F*	*O*	*S*
4th choice	*O*	*F*	*O*
5th choice	*S*	*R*	*R*

Example 3 Below is a preference schedule for Mr. Tucker's kindergarten class when they ranked their favorite colors among the choices Red, Blue and Purple.



57 votes

Who is the plurality winner? R 20 B 24 P 13

Blue

Is the plurality winner a majority winner? no

$\frac{1}{2}(57) = 28.5$ maj ≥ 29

Plurality with Elimination (also called sequential run-off)

- carried out in rounds
- After each round of voting the candidate (or alternative) with the fewest first place votes is eliminated and a new round of voting is done with the remaining candidates
- When only two candidates remain in a round, the candidate with the most votes wins the election
- Note: At any time during this process if a candidate has a majority of first-place votes, then that candidate is the winner
- For an N candidate election, Plurality with Elimination requires N-1 rounds

Example 4 The mayor of Smallville is being chosen in a Plurality with Elimination election. The four candidates are Paul (the town barber), Rita (head of the town council), Sarah (Superintendent of Education), and Tim (former District Attorney).

500 registered voters cast their preference ballots. The results are summarized in the preference schedule below.

Place	# of Voters: 130	120	100	150
1st	~~P~~	T	T	S
2nd	~~R~~	~~R~~	~~R~~	~~R~~
3rd	S	S	~~P~~	~~P~~
4th	T	~~P~~	S	T

majority >250

A. How many rounds will it take to determine a winner? 3

B. Who is the winner using Plurality with Elimination?

Round 1: P 130, R 0, S 150, T 220 → eliminate R

Round 2: P 130, S 150, T 220 → eliminate P

Round 3: S 280, T 220 → Sarah wins

Example 5

Preference	12 voters	9 voters	3 voters	8 voters
1st	Charles	~~Bonnie~~	Adam	Adam
2nd	Adam	Charles	Charles	~~Bonnie~~
3rd	~~Bonnie~~	Adam	~~Bonnie~~	Charles

total 32 votes
majority >16

A. How many rounds will it take to determine a winner? 2

B. Who is the winner using Plurality with Elimination?

Round 1: A 11, B 9, C 12 → elim. B

Round 2: A 11, C 21 → Charles wins

Example 6 Who is the Plurality with Elimination winner?

Preference	15 voters	7 voters	13 voters	5 voters	2 voters
1st	Helen	~~Eddie~~	Grover	~~Donna~~	~~Eddie~~
2nd	~~Donna~~	~~Donna~~	~~Flora~~	Grover	~~Flora~~
3rd	~~Eddie~~	Grover	~~Eddie~~	~~Flora~~	Grover
4th	~~Flora~~	~~Flora~~	~~Donna~~	~~Eddie~~	~~Donna~~
5th	Grover	Helen	Helen	Helen	Helen

42 votes
majority → 21

Round 1

D 5
E 9
F 0 elim. F
G 13
H 15

Round 2

D 5
E 9 elim D
G 13
H 15

Round 3

E 9 elim E
G 18
H 15

Round 4

G 27 Grover wins
H 15

Borda Count

- if there are m candidates, then for each vote, m points are assigned to the 1st choice, (m – 1) points are assigned to the 2nd choice, and so on
- the candidate that receives more points in total than any other is declared the winner
- invented in 1770 by the French mathematician and physicist Jean-Charles Borda

Advantages

- Incorporates all information from a preference ballot
- takes candidate with best average ranking
- preferable when comparing a large number of candidates

Example 7 The Math Appreciation Society is electing its president. The candidates are Alisha (A), Boris (B), Carmen (C), and Dave (D). Each of the 37 members votes with a preference ballot. Who should be the winner using a Borda Count?

Number of Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A