

# Calculus AB

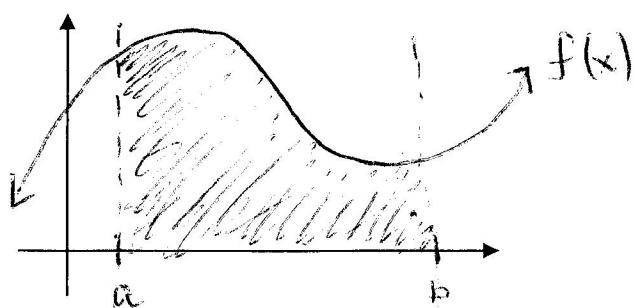
## Notes: Definite Integrals

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### Definite Integral

If  $f$  is continuous and non-negative on the closed interval  $[a, b]$  then the area of the region bounded by the graph of  $f$ , the  $x$ -axis and the vertical lines  $x = a$  and  $x = b$  is given by:

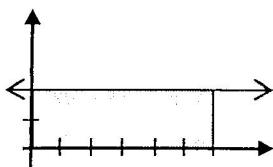
$$\text{area} = \int_a^b f(x) dx$$



Ex 1: Find the area.

A.  $\int_0^6 2dx$

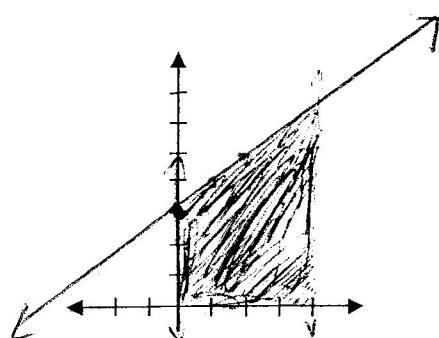
$$(6)(2) = \boxed{12}$$



B.  $\int_0^4 (x + 3) dx$

$$\frac{1}{2}(4)(3+7)$$

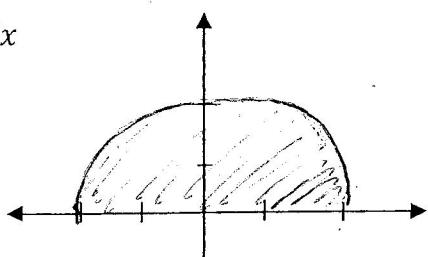
$$\boxed{20}$$



C.  $\int_{-2}^2 \sqrt{4 - x^2} dx$

$$\frac{1}{2}\pi(2)^2$$

$$\boxed{2\pi}$$



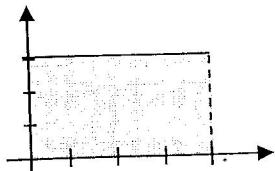
$$y = \sqrt{4 - x^2}$$

$$y^2 = 4 - x^2$$

$$x^2 + y^2 = 4$$

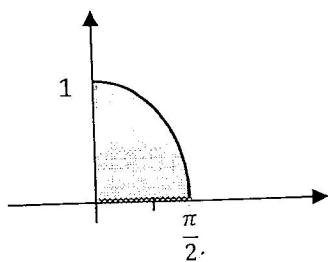
Ex 2: Write an integral that represents each of the following areas.

A.



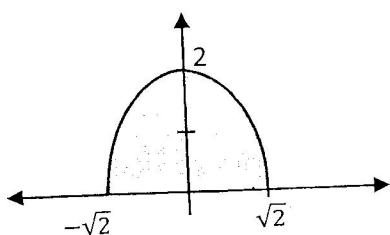
$$\int_0^4 3 \, dx$$

B.



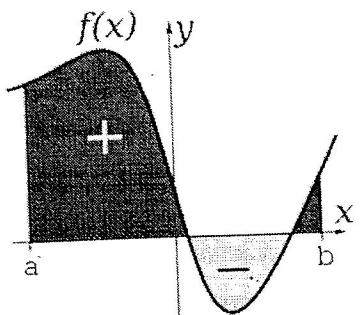
$$\int_0^{\sqrt{2}} \cos x \, dx$$

C.



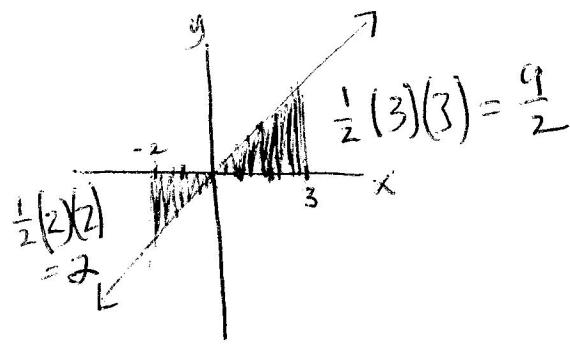
$$\int_{-\sqrt{2}}^{\sqrt{2}} (-x^2 + 2) \, dx$$

- If a graph falls below the x-axis that portion is considered "negative area" when evaluating the integral



Ex 3: Evaluate.

$$\int_{-2}^3 x \, dx$$
$$-\frac{2}{2} + \frac{9}{2}$$
$$\boxed{\frac{5}{2}}$$



## Properties of Integrals

$$1. \int_a^a f(x)dx = 0$$

$$2. \int_b^a f(x)dx = - \int_a^b f(x)dx$$

$$3. \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$4. \int_a^b kf(x)dx = k \int_a^b f(x)dx$$

$$5. \int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

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Ex 4: Given  $\int_2^4 x^3 dx = 60$ ,  $\int_2^4 x dx = 6$ , &  $\int_2^4 dx = 2$

find:

A.  $\int_2^2 x^3 dx = C$

B.  $\int_2^4 15 dx = 15 \int_2^4 dx = 15(2) = 30$

C.  $\int_2^4 (x^3 + 4) dx = \int_2^4 x^3 dx + 4 \int_2^4 dx = 60 + 4(2) = 68$

D.  $\int_2^4 (6 + 2x - x^3) dx = 6 \int_2^4 dx + 2 \int_2^4 x dx - \int_2^4 x^3 dx$   
 $= 6(2) + 2(6) - 60$   
 $= -36$