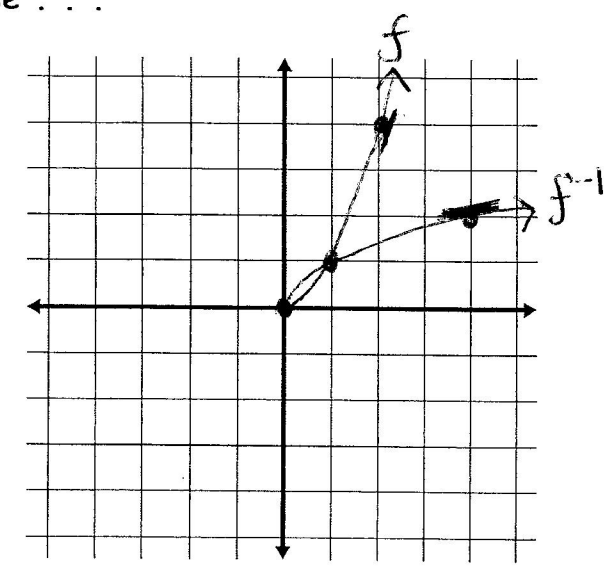
Developing the Idea of the Derivative of an Inverse . . .

A. Graph $f(x) = x^2$ for $x \geq 0$. range $y \geq 0$



B. Find the slope of the tangent line at x = 2.

$f'(x) = 2x$

$f'(2) = 4$

C. Find the inverse of $f(x)$.

$x = y^2$

$y = \pm\sqrt{x}$ $\quad f^{-1}(x) = \sqrt{x}, \; x \geq 0$

D. Find the derivative of the inverse.

$$\left(f^{-1}(x)\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

E. Find the slope of the tangent line at x = 4. for the inverse.

$$\left(f^{-1}(4)\right)' = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

F. Compare the value of $\left((f^{-1})(4)\right)'$ to that of $f'(2)$. What is the relationship?

they're reciprocals !!!

- The derivative of an inverse evaluated at $f(a)$ (y-value of the original) is equal to the reciprocal of $f'(x)$ evaluated at $x = a$. (x-value)

- If g is the inverse of f, then $g'(x) = \dfrac{1}{f'(g(x))} = \dfrac{1}{f'(f^{-1}(x))}$

Notes – Derivatives of Inverse Functions

Review: *How do you find the inverse of a function?* switch x & y, solve for y

What is a one-to-one function? inverse is a function (check using HLT)

Let f be a one-to-one differentiable function with inverse function f^{-1}.
If $y = f^{-1}(x)$ so that $x = f(y)$, then $dy/dx = 1/(dx/dy)$ with $dy/dx \neq 0$.

Ex1) If $f(x) = x^3 + 2x - 10$, find $(f^{-1})'(x)$

inverse: $x = y^3 + 2y - 10$

deriv: $1 = 3y^2 \frac{dy}{dx} + 2\frac{dy}{dx}$

$1 = \frac{dy}{dx}(3y^2 + 2)$

$\frac{dy}{dx} = \frac{1}{3y^2+2}$

Ex2) If $f(x) = 2x^5 + x^3 + 1$, find
(a) $f(1)$ & $f'(1)$ **(b)** $(f^{-1})(4)$ & $(f^{-1})'(4)$

$f(1) = 2(1)^5 + (1)^3 + 1 = 4$

$f'(x) = 10x^4 + 3x^2$

$f'(1) = 10 + 3 = 13$

$f: (1, 4)$

$f^{-1}: (4, 1)$

$f^{-1}(4) = 1$

$(f^{-1})'(4) = \frac{1}{13}$

Ex3) If $f(x) = 5x^3 + x + 8$, find $(f^{-1})'(8)$.

$f: (0, 8)$

$f^{-1}: (8, 0)$

$8 = 5x^3 + x + 8$

$0 = 5x^3 + x$

$0 = x(5x^2 + 1)$

$x = 0$ $\quad 5x^2 + 1 = 0$

$x^2 = -\frac{1}{5}$

$f'(x) = 15x^2 + 1$

$f'(0) = 1$

$(f^{-1})'(8) = \frac{1}{1} = \boxed{1}$

Ex4) If $f(2) = -3$, $f'(2) = 3/4$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

$f: (2, -3)$ $\quad f^{-1}: (-3, 2)$ point.

$g'(-3) = (f^{-1}(-3))' = \frac{4}{3}$ slope

$f'(2) = \frac{3}{4}$

$y - 2 = \frac{4}{3}(x + 3)$

Ex5) Let $h(x) = xg(x)$, where $g(x) = f^{-1}(x)$. Use the table of values below to find $h'(5)$.

x	$f(x)$	$f'(x)$
2	4	–1
3	5	2
5	1	3

$f: (3, 5)$

$g: (5, 3)$

(a) ½ (b) 2.5 (c) 3 (d) 4.667 (e) 5.5

$h'(x) = x \cdot g'(x) + g(x) \cdot 1$

$h'(5) = 5 \cdot g'(5) + g(5)$

$5(\frac{1}{2}) + 3$

5.5

$g'(5) = (f^{-1})'(5) = \frac{1}{2}$ $\quad f'(3) = 2$

deriv. of orig. at an x-value, take recip = deriv. of inv. at the y-value of the orig. pt (x-value of the inver