

# Differential Equations - Day 2

12/5/18

EX1 Find the general solution.

$$\textcircled{A} \quad \frac{dy}{dx} = \frac{x^2 + 2}{3y^2}$$

$$\int y^2 dy = \int \frac{x^2 + 2}{3} dx$$

$$\frac{1}{3}y^3 = \frac{1}{9}x^3 + \frac{2}{3}x + C$$

$$y^3 = \frac{1}{3}x^3 + 2x + C$$

$$y = \sqrt[3]{\frac{1}{3}x^3 + 2x + C}$$

$$\textcircled{B} \quad yy' - 2e^x = 0$$

$$y \frac{dy}{dx} = 2e^x$$

$$\int y dy = \int 2e^x dx$$

$$\frac{1}{2}y^2 = 2e^x + C$$

$$y^2 = 4e^x + C$$

$$y = \pm \sqrt{4e^x + C}$$

EX2 Given  $xydx + e^{-x^2}(y^2 - 1)dy = 0$ . Find the particular soln. if  $y(0) = 1$ .

$$e^{-x^2}(y^2 - 1)dy = -xydx$$

$$\int \frac{y^2 - 1}{y} dy = \int \frac{-x}{e^{-x^2}} dx$$

$$\int \left(y - \frac{1}{y}\right) dy = \int -xe^{x^2} dx$$

$$\frac{1}{2}y^2 - \ln|y| = -\frac{1}{2}e^{x^2} + C$$

$$\frac{1}{2}(1)^2 - \ln|1| = -\frac{1}{2}e^0 + C$$

$$\frac{1}{2} - 0 = -\frac{1}{2} + C$$

$$C = 1$$

$$\frac{1}{2}y^2 - \ln|y| = -\frac{1}{2}e^{x^2} + 1$$

$$u = x^2$$

$$\frac{du}{dx} = 2x$$

$$\frac{1}{2}du = xdx$$

$$-\frac{1}{2} \int e^u du = -\frac{1}{2}e^u + C$$

$$y^2 - \ln y^2 = -e^{x^2} + 2$$

EX3 Given  $y\sqrt{1-x^2}y' - x\sqrt{1-y^2} = 0$  and  $(0,1)$ .  
Find the particular soln.

$$y\sqrt{1-x^2}\frac{dy}{dx} = x\sqrt{1-y^2}$$

$$\int \frac{y}{\sqrt{1-y^2}} dy = \int \frac{x}{\sqrt{1-x^2}} dx$$

$$u = 1-y^2$$

$$-\sqrt{1-y^2} = -\sqrt{1-x^2} + C$$

$$\frac{du}{dy} = -2y$$

$$-\sqrt{1-1^2} = -\sqrt{1-0^2} + C$$

$$-\frac{1}{2}du = ydy$$

$$0 = -1 + C$$

$$1 = C$$

$$-\frac{1}{2} \int \frac{du}{\sqrt{u}} = -\frac{1}{2} \int u^{-\frac{1}{2}} du$$

$$-\sqrt{1-y^2} = -\sqrt{1-x^2} + 1$$

$$-1u^{\frac{1}{2}}$$

$$\sqrt{1-y^2} = \sqrt{1-x^2} - 1$$

$$1-y^2 = (\sqrt{1-x^2} - 1)^2$$

$$-y^2 = (\sqrt{1-x^2} - 1)^2 - 1$$

$$y^2 = -(\sqrt{1-x^2} - 1)^2 + 1$$

$$y = \pm \sqrt{-(\sqrt{1-x^2} - 1)^2 + 1}$$

$$y = \sqrt{-(\sqrt{1-x^2} - 1)^2 + 1}$$