

Exponential Functions

$$y = e^x$$

domain: $(-\infty, \infty)$

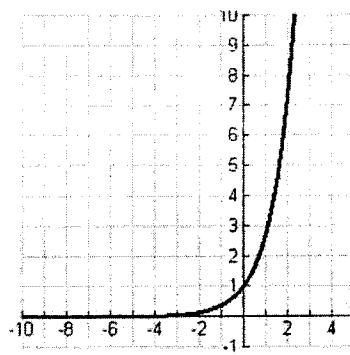
range: $(0, \infty)$

continuous on its domain

always increasing (strictly monotonic)

concave up

one-to-one



Derivative Rules

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Integration Rules

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Example 1 Find the derivative:

A. $f(x) = e^{4x^2 - 6}$

$$f'(x) = e^{4x^2 - 6} \cdot 8x = 8x e^{4x^2 - 6}$$

B. $f(x) = e^{\frac{3}{x}}$

$$f'(x) = e^{-\frac{3}{x}} \cdot -\frac{3}{x^2} = \frac{-3}{x^2 e^{\frac{3}{x}}}$$

C. $y = x^7 e^x$

$$y' = x^7 \cdot e^x + e^x \cdot 7x^6 = x^6 e^x (x + 7)$$

$$D. ye^x + 2x - 3y^2 = 10$$

$$y \cdot e^x + e^x \cdot 1 \frac{dy}{dx} + 2 - 6y \frac{dy}{dx} = 0$$

$$e^x \frac{dy}{dx} - 6y \frac{dy}{dx} = -ye^x - 2$$

$$\frac{dy}{dx} (e^x - 6y) = -ye^x - 2 \quad \frac{dy}{dx} = \frac{-ye^x - 2}{e^x - 6y}$$

$$E. y = 3^{4x}$$

$$y' = 3^{4x} \cdot \ln 3 \cdot 4$$

Example 2 Find:

$$A. \int e^{3x+1} dx$$

$$u = 3x+1 \quad \frac{du}{dx} = 3 \quad \frac{1}{3} \int e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{3x+1} + C$$

$$\frac{1}{3} du = dx$$

$$B. \int \frac{e^x}{x^2} dx = \int e^x \cdot \frac{1}{x^2} dx$$

$$u = \frac{1}{x} = x^{-1}$$

$$\frac{du}{dx} = -1x^{-2}$$

$$-du = \frac{1}{x^2} dx$$

$$-\int e^u du = -e^u + C = -e^{\frac{1}{x}} + C$$

$$C. \int 5xe^{-x^2} dx$$

$$u = -x^2$$

$$\frac{du}{dx} = -2x$$

$$-\frac{1}{2} du = x dx$$

$$5 \cdot -\frac{1}{2} \int e^u du = -\frac{5}{2} e^u + C = -\frac{5}{2} e^{-x^2} + C = -\frac{5}{2e^{x^2}} + C$$

$$D. \int_0^1 \frac{e^x}{1+e^x} dx$$

$$u = 1 + e^x$$

$$\frac{du}{dx} = e^x$$

$$du = e^x dx$$

$$\int \frac{du}{u} = \ln|u| + C = \ln|1 + e^x| + C \Big|_0^1$$

$$= \ln|1 + e^1| + C - (\ln|1 + e^0| + C)$$

$$= \ln(1 + e) - \ln(2) = \ln \frac{1 + e}{2}$$

$$E. \int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx = \int \frac{2(e^x - e^{-x})}{(e^x + e^{-x})^2} dx$$

$$u = e^x + e^{-x}$$

$$\begin{aligned}\frac{du}{dx} &= e^x - e^{-x} \\ du &= (e^x - e^{-x}) dx\end{aligned}$$

$$2 \int \frac{du}{u^2} = 2 \int u^{-2} du = -2u^{-1} + C = \frac{-2}{e^x + e^{-x}} + C$$

$$F. \int 3^x dx$$

$$\frac{3^x}{\ln 3} + C$$

Example 3 Find an equation of the tangent line to the graph of $f(x) = e^{3x} \ln x$ at the point $(1, 0)$.

$$f'(x) = e^{3x} \cdot \frac{1}{x} + \ln x \cdot e^{3x} \cdot 3$$

$$f'(1) = e^3 \cdot 1 + \ln 1 \cdot e^3 \cdot 3 = e^3$$

$$y - 0 = e^3(x - 1)$$

Example 4 Find the particular solution that satisfies the conditions $f''(x) = \sin x + e^{2x}$, $f(0) = \frac{1}{4}$, and $f'(0) = \frac{1}{2}$.

$$f'(x) = \int (\sin x + e^{2x}) dx = -\cos x + \frac{1}{2} e^{2x} + C$$

$$\int e^{2x} dx$$

$$u = 2x \quad \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$$

$$\frac{du}{dx} = 2$$

$$\frac{1}{2} du = dx$$

$$\frac{1}{2} = -\cos 0 + \frac{1}{2} e^{2(0)} + C$$

$$\frac{1}{2} = -1 + \frac{1}{2} + C$$

$$1 = C$$

$$f'(x) = -\cos x + \frac{1}{2} e^{2x} + 1$$

$$f(x) = \int f'(x) dx = -\sin x + \frac{1}{4} e^{2x} + x + C$$

$$\frac{1}{4} = -\sin 0 + \frac{1}{4} e^{2(0)} + 0 + C$$

$$0 = C$$

$$\boxed{f(x) = -\sin x + \frac{1}{4} e^{2x} + x} \quad B$$