

Calculus AB

Notes: The Fundamental Theorem of Calculus

Review: Suppose $\int_{-4}^3 f(x)dx = 9$, $\int_3^5 f(x)dx = -11$, $\int_{-4}^3 h(x)dx = 14$. Find:

$$a) \int_5^3 f(x)dx = - \int_3^5 f(x)dx$$

$$= -(-11)$$

$$\boxed{11}$$

$$c) \int_{-4}^3 [3f(x) - 4h(x)]dx =$$

$$3(9) - 4(14)$$

$$27 - 56$$

$$\boxed{-29}$$

$$b) \int_{-4}^5 f(x)dx =$$

$$\int_{-4}^3 f(x)dx + \int_3^5 f(x)dx = 9 + (-11)$$

$$\boxed{-2}$$

$$d) \int_3^{-4} f(x)dx =$$

$$- \int_{-4}^3 f(x)dx$$

$$\boxed{-9}$$

Fundamental Theorem of Calculus: — tells you how to evaluate a

If a function f is continuous on $[a, b]$ and F is the antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

definite
integral

Ex 1: Evaluate.

$$A. \int_2^5 (-3x + 4)dx = -\frac{3}{2}x^2 + 4x + C \Big|_2^5$$

$$= \left(-\frac{3}{2}(5)^2 + 4(5) + C \right) - \left(-\frac{3}{2}(2)^2 + 4(2) + C \right)$$

$$= \frac{-75}{2} + 20 + C + 6 - 8 - C = \boxed{\frac{-39}{2}}$$

$$B. \int_{-2}^{-1} \left(u - \frac{1}{u^2} \right) du = \frac{1}{2}u^2 + u^{-1} + C \Big|_{-2}^{-1}$$

$$\hookrightarrow u^{-2}$$

$$= \left(\frac{1}{2}(-1)^2 + (-1)^{-1} + C \right) - \left(\frac{1}{2}(-2)^2 + (-2)^{-1} + C \right)$$

$$= \frac{1}{2} - 1 + C - 2 + \frac{1}{2} - C = \boxed{-2}$$

$$C. \int_0^2 (t^{\frac{1}{2}} - 2)\sqrt{t} dt$$

$$\int_0^2 (t^{\frac{3}{2}} - 2t^{\frac{1}{2}}) dt$$

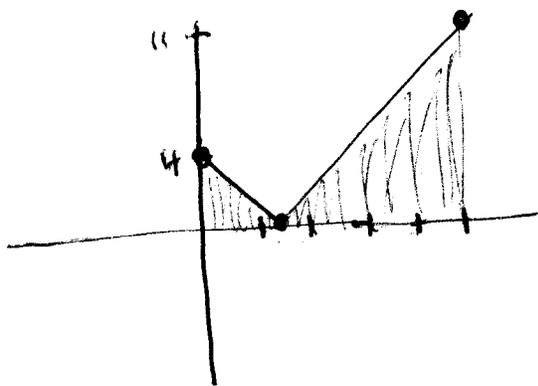
$$\frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C \Big|_0^2 = \frac{2}{5}(2)^{\frac{5}{2}} - \frac{4}{3}(2)^{\frac{3}{2}} + C - (0 - 0 + C)$$

$$= \boxed{\frac{2}{5}(2)^{\frac{5}{2}} - \frac{4}{3}(2)^{\frac{3}{2}}}$$

$$\begin{aligned}
 \text{D. } \int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}} dx &= \int_{-8}^{-1} \left(\frac{1}{2} X^{\frac{2}{3}} - \frac{1}{2} X^{\frac{5}{3}} \right) dx \\
 &= \left. \frac{3}{10} X^{\frac{5}{3}} - \frac{3}{16} X^{\frac{8}{3}} + C \right|_{-8}^{-1} \\
 &= \frac{3}{10} (-1)^{\frac{5}{3}} - \frac{3}{16} (-1)^{\frac{8}{3}} + C - \left(\frac{3}{10} (-8)^{\frac{5}{3}} - \frac{3}{16} (-8)^{\frac{8}{3}} + C \right) \\
 &= -\frac{3}{10} - \frac{3}{16} + C + \frac{48}{5} + 48 - C = \boxed{\frac{4569}{80}}
 \end{aligned}$$

$$\begin{aligned}
 \text{E. } \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 - \csc^2 x) dx &= 2x + \cot x + C \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
 &= 2\left(\frac{\pi}{2}\right) + \cot \frac{\pi}{2} + C - \left(\frac{\pi}{2} + \cot \frac{\pi}{4} + C \right) \\
 &= \pi + 0 + C - \frac{\pi}{2} - 1 - C \\
 &= \boxed{\frac{\pi}{2} - 1}
 \end{aligned}$$

$$\text{F. } \int_0^5 |3x - 4| dx$$

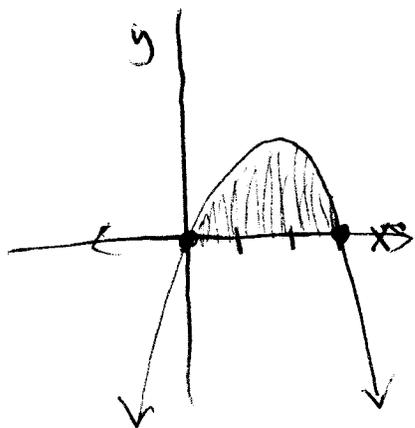


$$\frac{1}{2} \left(\frac{4}{3} \right) (4) + \frac{1}{2} \left(\frac{11}{3} \right) (11)$$

$$\frac{16}{6} + \frac{121}{6}$$

$$\boxed{\frac{137}{6}}$$

Ex 2: Find the area bounded by $y = -x^2 + 3x$ and $y = 0$.



$$\begin{aligned}
 y &= -x(x-3) \\
 -x &= 0 \quad x-3=0 \\
 x &= 0 \quad x=3
 \end{aligned}$$

$$\int_0^3 (-x^2 + 3x) dx = \left. -\frac{1}{3} x^3 + \frac{3}{2} x^2 + C \right|_0^3$$

$$\begin{aligned}
 &= \left(-\frac{1}{3} \cdot 27 + \frac{3}{2} \cdot 9 + C \right) - (0 + 0 + C) \\
 &= -9 + \frac{27}{2} + C - C = \boxed{\frac{9}{2}} \quad 15
 \end{aligned}$$