

Review of Integration

Calculus AP

1. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $[0, 80]$ seconds as shown in the table below. (2006 AB4/BC4)

t (sec)	0	10	20	30	40	50	60	70	80
$v(t)$ (ft/s)	5	14	22	29	35	40	44	47	49

- a) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight.
- b) Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$
2. A metal wire of length 8 centimeters (cm) is heated at one end. The table below gives selected values of the Temperature $T(x)$ in degrees Celsius, of the wire x cm from the heated end. The function T is decreasing and twice differentiable. (2005 AB3/BC3)

Distance, x (cm)	0	1	5	6	8
Temp. $T(x)$ °C	100	93	70	62	55

- a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with four subintervals indicated by the data in the table. Indicate units of measure.
3. Use trapezoids to estimate the definite integral of $f(x) = -100x^2 + 90x + 14$ from $x = 0.1$ to $x = 1$. Use 9 increments.
4. The velocity (in mph) of a Piper Club aircraft traveling due west is recorded every minute during the first 10 min after takeoff. Use the Trapezoidal Rule to estimate the distance traveled after 10 minutes.

t	0	1	2	3	4	5	6	7	8	9	10
$v(t)$	0	50	60	80	90	100	95	85	80	75	85

5. Approximate the area bounded by $f(x) = 4 - x^2$ and the x -axis using
- Left-hand rectangles and four equal subdivisions
 - Right-hand rectangles and four equal subdivisions
 - Midpoint Rectangles and four equal subdivisions
 - Approximate this area using a Trapezoidal Rule and four equal subdivisions
 - Verify that that the Left-hand approximation and the Right hand approximation give the same result as the Trapezoidal rule

Notes Integration Table Problems 11/9/18

#1 a) the distance in feet traveled by the rocket from 10 sec to 70 sec

b) width subinterval = $\frac{70-10}{3} = 20$

$$20(22 + 35 + 44) = 20(101) = 2020 \text{ ft}$$

#2 a) (6, 62) (8, 55)

$$\frac{55-62}{8-6} = \frac{-7}{2} \frac{^{\circ}\text{C}}{\text{cm}}$$

b) avg. value = $\frac{1}{b-a} \int f(x) dx$

$$\frac{1}{8-0} \int_0^8 T(x) dx = \frac{1}{8}(605.5) = 75.6875 ^{\circ}\text{C}$$

trap: $\frac{1}{2}(1)(100+93)$
 $\frac{1}{2}(4)(93+70)$
 $\frac{1}{2}(1)(70+62)$
 $\frac{1}{2}(2)(62+55)$

605.5