Notes (Section 5.5)-Law of Sines

The Law of Sines (To be used when given 2 angles and a side or 2 consecutive sides and then an angle) (ASA, AAS, or SSA)

In any ΔABC with angles $A, B,$ and C opposite sides $a, b,$ and, c respectively, the following equation is true:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex 1: Solve ΔABC: A = 50°, B = 62°, a = 4.

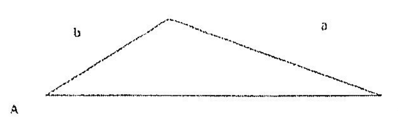
Ex 2: In triangle ΔPQR, m∠P = 36°, m∠Q=48°, and p=8.

The Ambiguous Case: When given the measures of 2 sides and the angle opposite one of those sides (SSA), it is possible that two, one or no triangle is formed by this angle and sides.

Here is a summary of possibilities:

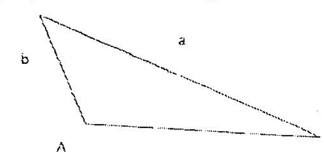
1. If the angle, A, is acute (less than 90^{o}) and the side opposite the angle, a, is greater than or equal to the other side, b:

There can only be one solution

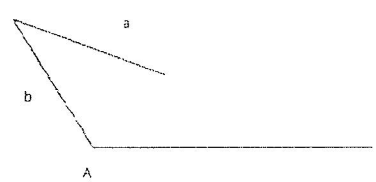


2. If the angle, A, is obtuse or right:

There is one solution if $a > b$



There is no solution if $a \leq b$



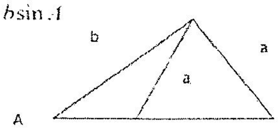
Ex 3: $\angle C = 97^{o}$, $c = 45$, $a = 39$

Ex 4: $\angle A = 47^{o}$, $a = 20$, $b = 12$

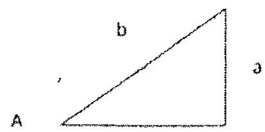
Ex 5: $\angle A = 114^{o}$, $a = 21$, $b = 32$

3. If the angle, A, is acute (less than 90^o), the height is bsinA and the side opposite the angle, a, is less than the other side, b:

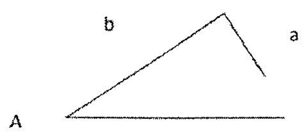
There are two solutions if $a > b\sin A$



There is one solution if $a = b\sin A$



There is no solution if $a < b\sin A$



Ex6: Solve ΔABC: $\angle A = 36°$, $b = 17$, $a = 16$

Ex7: Solve ΔABC: $\angle C = 82°$, $c = 12$, $b = 15$

Ex 8: A satellite passes over two tracking stations, *A* and ***B***, 100 km apart. When the satellite is between the two stations the angle of elevation at the stations are measured as 84.5° and 88.2° respectively. What is the distance between the satellite and station *A.?* How high is the satellite of the ground?

Ex 9: To find the distance across a river, a surveyor chooses point *A* and *B*, which are 200 ft. apart on one side of the river. She chooses a reference point *C* on the opposite side of the river and finds that $< BAC = 82°$ and $< ABC = 52°$. Find the distance across the river.

Notes (Section 5.6)-The Law of Cosines

The **Law of Cosines** states the ratio of the sine of an angle to the length its opposite angle is the same for all three angles.

In any ΔABC with angles $A, B,$ and$C$ opposite sides $a, b,$ and, c respectively, the following equation is true:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

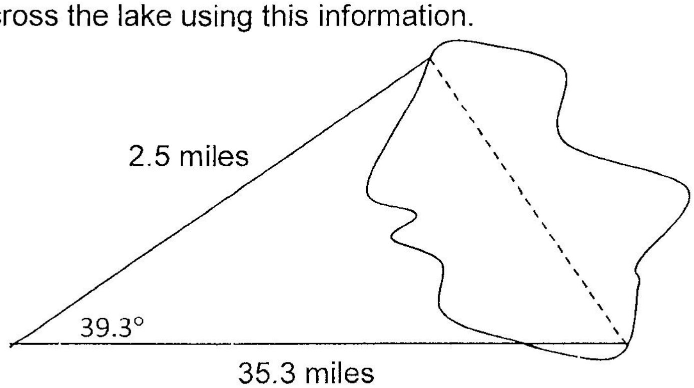
We can use the **Law of Cosines** to solve triangles when given ________ & ________.

Ex 1: Solve ΔABC: a = 10, b = 11, c = 17

Ex 2: Solve ΔABC: A = 35°, b = 43, c = 19

Ex 3: A girl is flying two kites at the same time. If the strings are 200 ft and 230 ft long and the kites are 110 ft apart, what angle do the strings make in her hand make?

Ex 4: To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake using this information.



Area of a triangle

ΔArea $= \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab\ sin\ C$

Ex1) Find the area of a triangle with sides of length 5 and 12 and included angle 40°.

Ex2) Find the area of Triangle ABC to the nearest thousandth if $c = 3.2$, $A = 16^0$, $B = 31^0$

Theorem Heron's Formula

Let $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$ be the sides of ΔABC, let $\boldsymbol{s}$ denote the **semi-perimeter;** $s = \frac{(a+b+c)}{2}$.

Then the area of ΔABC is given by **Area =** $\sqrt{\boldsymbol{s(s-a)(s-b)(s-c)}}$

Ex 3) Find the area of triangle ABC with $a = 2$, $b = 7$, $c = 8$

Ex 4) Find the area of an isosceles triangle with a perimeter of 39 and a base of length 17 inches.