

# L'Hopital's Rule

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Suppose that  $f(a) = 0 \neq g(a) = 0$ , that  $f'(a) \neq g'(a)$  exist, and  $g'(a) \neq 0$ .

Then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$ :

why it works...

$$\begin{aligned} \lim_{x \rightarrow a} \frac{f(x)}{g(x)} &= \lim_{x \rightarrow a} \frac{\overset{0}{f(x)-f(a)}}{\overset{0}{g(x)-g(a)}} = \lim_{x \rightarrow a} \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}} \\ &= \frac{\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}}{\lim_{x \rightarrow a} \frac{g(x)-g(a)}{x-a}} = \frac{f'(a)}{g'(a)} \end{aligned}$$

Find the limit.

①  $\lim_{x \rightarrow 0} \frac{\sin x}{x} \quad \frac{\sin 0}{0} = \frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{\cos x}{1} = \cos 0 = \boxed{1}$$

②  $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x} \quad \frac{3(0) - \sin 0}{0} = \frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{3 - \cos x}{1} = 3 - \cos 0 = 3 - 1 = \boxed{2}$$

$$\textcircled{3} \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} \quad \frac{\sqrt{1} - 1}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}(1) - 0}{1} = \frac{\frac{1}{2}(1)^{-\frac{1}{2}}}{1} = \boxed{\frac{1}{2}}$$

$$\textcircled{4} \quad \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} \quad \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2} \quad \frac{1 - \cos 0}{3(0)^2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{0 - (-\sin x)}{6x} \quad \frac{\sin 0}{6(0)} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\cos x}{6} = \frac{\cos 0}{6} = \boxed{\frac{1}{6}}$$

$$\textcircled{5} \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2} \quad \frac{\sqrt{1} - 1 - 0}{0^2} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}(1) - 0 - \frac{1}{2}}{2x} \quad \frac{\frac{1}{2}(1)^{-\frac{1}{2}} - \frac{1}{2}}{2(0)} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2} \cdot -\frac{1}{2}(1+x)^{-\frac{3}{2}}(1) - 0}{2} = \frac{-\frac{1}{4}(1)^{-\frac{3}{2}}}{2} = \boxed{-\frac{1}{8}}$$

## Indeterminate forms

$$\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{-\infty}, \frac{\infty}{-\infty}, \frac{-\infty}{\infty}$$

⑥  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} \quad \frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = \frac{0}{1} = \boxed{0}$$

not indeterminate

$$\infty + \infty = \infty$$

$$-\infty + \infty = -\infty$$

$$0^\infty = 0$$