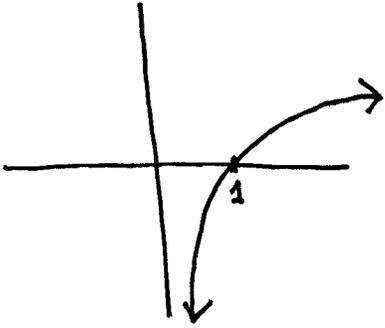


Natural Log Function $y = \ln x$

11/26/18



$$D: (0, \infty)$$

$$R: (-\infty, \infty)$$

continuous on its domain
always increasing
concave down

properties of logs

$$\textcircled{1} \ln 1 = 0$$

$$\textcircled{2} \ln e = 1$$

$$\textcircled{3} \ln(ab) = \ln a + \ln b$$

$$\textcircled{4} \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$\textcircled{5} \ln a^n = n \cdot \ln a$$

derivative rules

$$\frac{d}{dx} [\ln u] = \frac{1}{u} \cdot u' = \frac{u'}{u}$$

$$\frac{d}{dx} [\ln |u|] = \frac{u'}{u}$$

EX1 Find the derivative.

(A) $f(x) = \ln\left(\frac{x}{x+1}\right) = \ln x - \ln(x+1)$

$$\begin{aligned} f'(x) &= \frac{1}{\frac{x}{x+1}} \cdot \frac{(x+1)(1) - (x)(1)}{(x+1)^2} \\ &= \frac{x+1}{x} \cdot \frac{1}{(x+1)^2} \\ &= \frac{1}{x(x+1)} \end{aligned} \quad \left. \vphantom{\begin{aligned} f'(x) &= \frac{1}{\frac{x}{x+1}} \cdot \frac{(x+1)(1) - (x)(1)}{(x+1)^2} \\ &= \frac{x+1}{x} \cdot \frac{1}{(x+1)^2} \\ &= \frac{1}{x(x+1)} \end{aligned}} \right\} \begin{aligned} f'(x) &= \frac{1}{x} - \frac{1}{x+1} \\ &= \frac{x+1 - x}{x(x+1)} \\ &= \frac{1}{x(x+1)} \end{aligned}$$

(B) $y = \ln \sqrt{\frac{x-1}{x+1}} = \ln \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}} = \frac{1}{2} \ln \left(\frac{x-1}{x+1}\right)$

$$= \frac{1}{2} [\ln(x-1) - \ln(x+1)]$$
$$y' = \frac{1}{2} \left[\frac{1}{x-1} - \frac{1}{x+1} \right] = \frac{1}{2} \left[\frac{x+1 - (x-1)}{(x-1)(x+1)} \right] = \frac{1}{2} \left[\frac{2}{(x-1)(x+1)} \right]$$
$$= \frac{1}{x^2 - 1}$$

(C) $y = \ln|\sec x|$

$$y' = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x$$

Logarithmic Differentiation - use when you have a function raised to a function OR a complicated rational expression

- ① change $f(x)$ to y and take the \ln of both sides
- ② use properties of logs to expand the right side
- ③ take the derivative w.r.t. x on both sides
- ④ solve for $\frac{dy}{dx}$
- ⑤ replace "y" with original eqn.

EX 2

① Find $f'(x)$ if $f(x) = \frac{(x+1)(x+2)}{(x-1)(x-2)}$

$$\ln y = \ln \left(\frac{(x+1)(x+2)}{(x-1)(x-2)} \right)$$

$$\ln y = \ln[(x+1)(x+2)] - \ln[(x-1)(x-2)]$$

$$\ln y = \ln(x+1) + \ln(x+2) - \ln(x-1) - \ln(x-2)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}$$

$$\frac{dy}{dx} = y \left(\nearrow \right) = \frac{(x+1)(x+2)}{(x-1)(x-2)} \left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2} \right)$$

(B) $g(x) = x^{\sin x}$ Find $g'(x)$.

$$\ln y = \ln x^{\sin x}$$

$$\ln y = (\sin x) \cdot \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = (\sin x) \left(\frac{1}{x} \right) + (\ln x) (\cos x)$$

$$\frac{dy}{dx} = y \left(\nearrow \right) = x^{\sin x} \left(\frac{\sin x}{x} + (\ln x) (\cos x) \right)$$

(C) $f(x) = \frac{(x^2+3)^3 (4x-3)}{\sqrt{x^4+6}}$ Find $f'(x)$.

$$\ln y = \ln \left(\frac{(x^2+3)^3 (4x-3)}{\sqrt{x^4+6}} \right)$$

$$\ln y = \ln [(x^2+3)^3 (4x-3)] - \ln \sqrt{x^4+6}$$

$$\ln y = \ln (x^2+3)^3 + \ln(4x-3) - \ln (x^4+6)^{1/2}$$

$$\ln y = 3 \ln (x^2+3) + \ln(4x-3) - \frac{1}{2} \ln(x^4+6)$$

$$\frac{1}{y} \frac{dy}{dx} = 3 \cdot \frac{1}{x^2+3} \cdot 2x + \frac{1}{4x-3} \cdot 4 - \frac{1}{2} \cdot \frac{1}{x^4+6} \cdot 4x^3$$

$$\frac{dy}{dx} = y \left(\nearrow \right)$$

$$\frac{dy}{dx} = \frac{(x^2+3)^3 (4x-3)}{\sqrt{x^4+6}} \left(\frac{6x}{x^2+3} + \frac{4}{4x-3} - \frac{2x^3}{x^4+6} \right)$$