

Notes – PVA

Motion along a Line – Suppose that an object is moving along a coordinate line (say the x -axis) so that we know its position s on that line as a function of time t : $s = f(t)$

The displacement of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$ and the **average velocity** of the object over that time interval is $\frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$

If we have a position function, the first derivative will give us velocity (instantaneous), and the second derivative will give us acceleration.

DEFINITION	SYMBOLIC RULE	
Instantaneous Velocity – is the derivative of the position function with respect to time.	$v(t) = \frac{ds}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$	The earliest questions that motivated the discovery of calculus were concerned with velocity & acceleration, particularly of freely falling bodies under the force of gravity.
Speed – is the absolute value of velocity.	$\text{speed} = v(t) = \left \frac{ds}{dt} \right $	The mathematical description of this type of motion captured the imagination of many great scientists, including Aristotle, Galileo, and Newton. Experimental and theoretical investigations revealed that the distance a body released from rest falls freely is proportional to the square of the amount of time it has fallen. We express this by saying: $s = \frac{1}{2}gt^2$
Acceleration – is the derivative of velocity with respect to time.	$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	FREE FALL CONSTANTS (on Earth) English units: $g = 32 \text{ ft/sec}^2$ $s(t) = \frac{1}{2}(32)t^2 = 16t^2$ Metric units: $g = 9.8 \text{ m/sec}^2$ $s(t) = \frac{1}{2}(9.8)t^2 = 4.9t^2$

<u>English</u>	<u>Calculus</u>
particle at rest	velocity = 0
particle moving right	velocity > 0
particle moving left	velocity < 0
particle changes direction	velocity changes sign
total distance traveled from time t_1 to t_2 (where t_c = time when the particle changes direction)	$ s(t_1) - s(t_c) + s(t_c) - s(t_2) $

Function	Units	Examples / Abbreviations
$s(t)$	linear units	feet or meters: ft or m
$v(t)$	linear units per unit of time	feet per second: ft/s meters per second: m/s
$a(t)$	linear units per unit of time squared	feet per second squared: ft/s ² centimeters per second squared: cm/s ²

P

- ∞ Particle moves forward when velocity is positive.
- ∞ Particle moves backward when velocity is negative.
- ∞ Particle standing still or at rest when graph of position is horizontal.
(think – velocity is zero)
- ∞ Particle moving right when graph of position has a positive slope.
- ∞ Particle moving left when graph of position has a negative slope.
- ∞ Particle moves at its greatest speed when absolute value of velocity is maximized.

V

- ∞ Particle's acceleration is zero when velocity is constant.
- ∞ Particle's acceleration is negative when velocity is decreasing.
- ∞ Particle's acceleration is positive when velocity is increasing.
- ∞ Particle speeds up when velocity is negative and decreasing or velocity is positive and increasing. *vel & accel have same sign*
- ∞ Particle slows down when velocity is positive and decreasing or velocity is negative and increasing. *vel & accel have different signs*

A

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PVA class notes

1. The position function of a particle moving along the x-axis is given by $x(t) = t^3 - 12t^2 + 36t - 20$ for $0 \leq t \leq 8$.

- Find the velocity and acceleration of the particle.
- Find the open interval when the particle is moving to the left. \rightarrow vel is neg
- Find the velocity of the particle when the acceleration is 0.
- Describe the motion of the particle.

a) $v(t) = x'(t) = 3t^2 - 24t + 36$

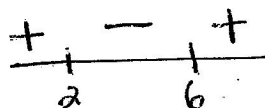
$a(t) = v'(t) = 6t - 24$

b) $3t^2 - 24t + 36 = 0$

$3(t^2 - 8t + 12) = 0$

$3(t-2)(t-6) = 0$

$t=2 \quad t=6$



(2,6)

c) $6t - 24 = 0$
 $t = 4$

$v(4) = 3(4)^2 - 24(4) + 36 = -12$

d) $t \mid x(t)$

0	-20
2	12
6	-20
8	12

 At $t=0$, the particle is 20 units to the left of the origin. The particle moves to the right for 32 units. At $t=2$, the particle moves to the left 32 units over 4 sec. At $t=6$ until $t=8$, the particle moves 32 units to the right.

2. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in meters.

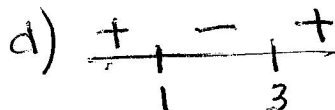
- Find the velocity at time t . $v(t) = 3t^2 - 12t + 9$ m/s
- What is the velocity at 2 s? at 4 s? $v(2) = 3(2)^2 - 12(2) + 9 = -3$ m/s
- When is the particle at rest? $vel = 0$ $v(4) = 3(4)^2 - 12(4) + 9 = 9$ m/s
- When is the particle moving to the right? vel is pos.
- Find the total distance traveled by the particle during the first 5 s.

c) $3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

$t = 3 \text{ sec} \quad t = 1 \text{ sec}$



(0,1) and (3,5)

e) $t \mid s(t)$

0. | 0

1. | 4

3. | 0

5. | 20

28 m

3. Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground.

$s(t) = -\frac{1}{2}gt^2 + v_0t + h_0$

- Write a height equation. $s(t) = -4.9t^2 + 450$
- Find the average velocity of the ball from $t = 3$ seconds to $t = 5$ seconds.
- Find the velocity of the ball at 5 seconds.
- When will the ball hit the ground? $ht = 0$
- With what velocity will the ball hit the ground?

b) $\text{avg. vel.} = \frac{\Delta \text{pos}}{\Delta \text{time}}$

$(3, 405.9) \quad (5, 327.5)$

$\frac{327.5 - 405.9}{5 - 3} = -39.2$

c) $v(t) = -9.8t$

$v(5) = -9.8(5) = -49$ m/s

d) $-4.9t^2 + 450 = 0$

$t^2 = \frac{4500}{4.9} \quad t = 9.583 \text{ sec}$

e) $v(9.583) = -9.8(9.583) = -93.915$ m/s