Related Rates Day 2 notes

Example 1
A ladder 26 feet long is resting against the side of a house. The foot of the ladder is pulled away at a rate of 4 ft/sec. How fast is the top of the ladder sliding down the wall when the base of the ladder is 10 feet from the wall?
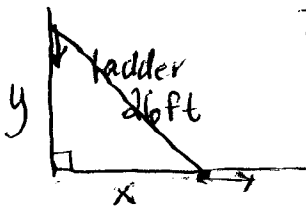
$$\frac{dx}{dt} = 4 \text{ ft/sec}$$

Find $\frac{dy}{dt}$ when $x = 10$ ft

$$x^2 + y^2 = 26^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(10)(4) + 2(24)\frac{dy}{dt} = 0$$

$$80 + 48\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-80}{48} = -\frac{5}{3} \text{ ft/sec}$$
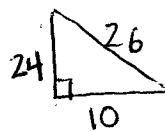
the ladder is sliding down at a rate of $5/3$ ft/sec when $x = 10$ ft

Example 2
Skippy and Binky are going to begin a hike at the same location and travel in perpendicular directions. Skippy travels due north at a rate of 5 mph; Binky travels due west at a rate of 8 mph. At what rate is the distance between them changing 3 hours into the hike?
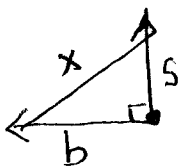
$$\frac{ds}{dt} = 5 \text{ mph}$$

$$\frac{db}{dt} = 8 \text{ mph}$$

Find $\frac{dx}{dt}$ when $t = 3$ hr
$S = 15$ mi $b = 24$ mi
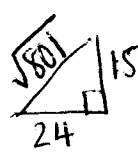
$$s^2 + b^2 = x^2$$

$$2s\frac{ds}{dt} + 2b\frac{db}{dt} = 2x\frac{dx}{dt}$$

$$2(15)(5) + 2(24)(8) = 2(\sqrt{801})\frac{dx}{dt}$$

$$534 = 2\sqrt{801}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{267}{\sqrt{801}} \text{ mph}$$

$$9.434$$

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Example 3

A woman 6 feet tall walks at a rate of 3 feet per second away from the light that is 45 feet tall. When she is 25 feet from the light, at what rate is the tip of her shadow moving? At what rate is the length of her shadow changing?

$$\frac{dx}{dt} = 3 \text{ ft/sec} \qquad \frac{dy}{dt} - \frac{dx}{dt} = \frac{45}{13} - 3 = \frac{6}{13} \text{ ft/sec}$$

Find $\frac{dy}{dt}$ and $\frac{d(y-x)}{dt}$ when $x = 25$ ft

$$\frac{45}{y} = \frac{6}{y-x} \qquad -39y = -45x \qquad \frac{dy}{dt} = \frac{15}{13} \frac{dx}{dt} = \frac{15}{13}(3)$$

$$6y = 45y - 45x \qquad y = \frac{45}{39}x = \frac{15}{13}x \qquad \frac{dy}{dt} = \frac{45}{13} \text{ ft/sec}$$

Example 4

A man 6 feet tall walks at a rate of 5 feet per second towards a light that is 25 feet tall. When he is 15 feet from the light, at what rate is the tip of his shadow moving? At what rate is the length of his shadow changing?

$$\frac{dx}{dt} = -5 \text{ ft/sec} \qquad \frac{dy}{dt} - \frac{dx}{dt} = \frac{-125}{19} - (-5) = \frac{-30}{19} \text{ ft/sec}$$

Find $\frac{dy}{dt}$ and $\frac{d(y-x)}{dt}$ when $x = 15$ ft

$$\frac{25}{y} = \frac{6}{y-x} \qquad -19y = -25x \qquad \frac{dy}{dt} = \frac{25}{19}\frac{dx}{dt} = \frac{25}{19}(-5)$$

$$6y = 25y - 25x \qquad y = \frac{25}{19}x \qquad \frac{dy}{dt} = -\frac{125}{19} \text{ ft/sec}$$

Example 5

A hot air balloon is rising at a rate of 9 yards per second. From a point 15 yards away on the ground, an observer measures the height of the balloon. What is the rate of change of the angle from the ground to the balloon when the balloon is 100 yards high?

$$\frac{dy}{dt} = 9 \text{ yd/sec}$$
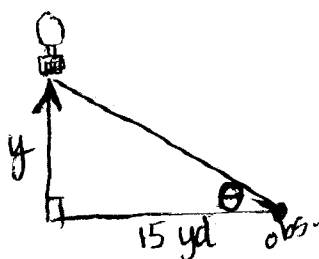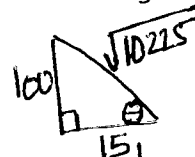
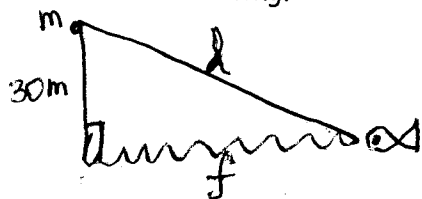Find $\frac{d\theta}{dt}$ when $y = 100$ yds

$$\tan\theta = \frac{y}{15}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{15}\frac{dy}{dt}$$

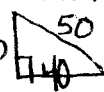$$\frac{d\theta}{dt} = \frac{\frac{1}{15}\frac{dy}{dt}}{\sec^2\theta}$$

$$\frac{d\theta}{dt} = \frac{\frac{1}{15}(9)}{\left(\frac{\sqrt{10225}}{15}\right)^2} = \frac{27}{2045} = .013 \text{ rad/sec}$$

Example 6

A fish is being reeled in a rate of 2 m/sec (that is, a fishing line is being shortened by 2 m/sec) by the Old Man by the Sea. If he is sitting 30 meters above the water, how fast is the fish moving through the water when the line is 50 meters long?

$$\frac{d\ell}{dt} = -2 \text{ m/sec}$$
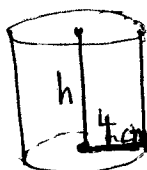
Find $\frac{df}{dt}$ when $\ell = 50$ m

$$30^2 + f^2 = \ell^2$$

$$2f\frac{df}{dt} = 2\ell\frac{d\ell}{dt}$$

$$\frac{df}{dt} = \frac{2\ell\frac{d\ell}{dt}}{2f} = \frac{(50)(-2)}{40}$$

$$\frac{df}{dt} = -\frac{5}{2} \text{ m/sec}$$

Example 7

The height of a cylinder with radius of 4 cm is increasing at a rate of 2 centimeters per minute. Find the rate of change of the volume of the cylinder with respect to the time when the height is 10 centimeters.

$$\frac{dh}{dt} = 2 \text{ cm/min}$$

Find $\frac{dV}{dt}$ when $h = 10$ cm.

$$V = \pi r^2 h = \pi (4)^2 h = 16\pi h$$

$$\frac{dV}{dt} = 16\pi \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = 16\pi (2) = 32\pi \frac{cm^3}{min}$$

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