

# Related Rates Day 2 notes

## Guidelines for Solving Related-Rate Problems

**Step 1:** Read the problem, really! You'd be amazed how many people skip this step. Then read it again! ☺

**Step 2:** Draw a diagram showing what's going on. Identify all relevant information and assign variables to what's changing. Use the general case (numbers for values that NEVER change in this situation, and variables for anything that is changing).

⚡: Related Rates usually involve motion ... any diagram you draw is like a still picture of what is occurring. Any part of your picture that NEVER changes can be labeled with a constant (or number), but any part of your picture that is in motion or is changing MUST be labeled with a variable!

In other words, if the radius of a circle is increasing and you are asked to find the rate of change in the area at the exact moment when the radius is 5 cm, then your diagram would be a circle, but you would NOT label the radius 5 because it is changing ... you would label the radius  $r$ .

**Step 3:** Find the equation that gives the relationship between the variables you just named in step 2. This is sometimes the hardest part, but most problems fall into three categories ... a triangle that you can use a trigonometric ratio (involving sides and angles), the Pythagorean theorem (involving all 3 sides of a right triangle), or a known formula like Area, Volume, Distance, etc.

**Step 4:** Find the particular information (values of variables at the exact moment you drew your diagram) for the problem and write it down, and list what you are looking for (normally this would be a derivative).

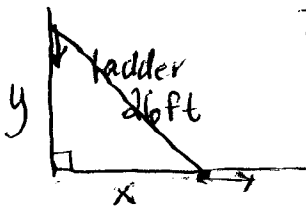
**Step 5:** Implicitly differentiate the equation with respect to time. Usually this equation will have at least two derivatives. If it has more than two, be sure you have enough information, or you may have find a relationship between two of the variables, and rewrite the equation in step 3 using this relationship.

**Step 6:** Plug in the particular information, and solve for the desired quantity. **DO NOT DO THIS UNTIL AFTER YOU HAVE TAKEN THE DERIVATIVE!**

**Step 7:** Write down your answer and circle it with your favorite color. (be sure to use correct units)

### Example 1

A ladder 26 feet long is resting against the side of a house. The foot of the ladder is pulled away at a rate of 4 ft/sec. How fast is the top of the ladder sliding down the wall when the base of the ladder is 10 feet from the wall?



$$\frac{dx}{dt} = 4 \text{ ft/sec}$$

Find  $\frac{dy}{dt}$  when  $x = 10 \text{ ft}$

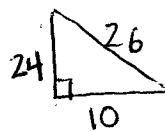
$$x^2 + y^2 = 26^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(10)(4) + 2(24) \frac{dy}{dt} = 0$$

$$80 + 48 \frac{dy}{dt} = 0$$

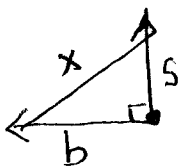
$$\frac{dy}{dt} = \frac{-80}{48} = -\frac{5}{3} \text{ ft/sec}$$



the ladder is sliding down at a rate of  $\frac{5}{3} \text{ ft/sec}$  when

### Example 2

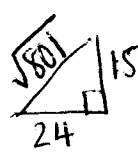
Skippy and Binky are going to begin a hike at the same location and travel in perpendicular directions. Skippy  $x = 10 \text{ ft}$  travels due north at a rate of 5 mph; Binky travels due west at a rate of 8 mph. At what rate is the distance between them changing 3 hours into the hike?



$$\frac{ds}{dt} = 5 \text{ mph}$$

$$\frac{db}{dt} = 8 \text{ mph}$$

Find  $\frac{dx}{dt}$  when  $t = 3 \text{ hr}$



$$s^2 + b^2 = x^2$$

$$2s \frac{ds}{dt} + 2b \frac{db}{dt} = 2x \frac{dx}{dt}$$

$$2(15)(5) + 2(24)(8) = 2(\sqrt{801}) \frac{dx}{dt}$$

$$534 = 2\sqrt{801} \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{267}{\sqrt{801}} \text{ mph} \approx 9.434$$

$$s = 15 \text{ mi} \quad b = 24 \text{ mi}$$

### Example 3

A woman 6 feet tall walks at a rate of 3 feet per second away from the light that is 45 feet tall. When she is 25 feet from the light, at what rate is the tip of her shadow moving? At what rate is the length of her shadow changing?

$$\frac{dx}{dt} = 3 \text{ ft/sec}$$

$$\frac{dy}{dt} - \frac{dx}{dt} = \frac{45}{13} - 3 = \frac{6}{13} \text{ ft/sec}$$

Find  $\frac{dy}{dt}$  and  $\frac{d(y-x)}{dt}$  when  $x=25$  ft

$$\frac{45}{y} = \frac{6}{y-x}$$

$$-39y = -45x$$

$$\frac{dy}{dt} = \frac{15}{13} \frac{dx}{dt} = \frac{15}{13} (3)$$

$$y = \frac{45}{39}x = \frac{15}{13}x$$

$$\frac{dy}{dt} = \frac{45}{13} \text{ ft/sec}$$

$$6y = 45y - 45x$$

A man 6 feet tall walks at a rate of 5 feet per second towards a light that is 25 feet tall. When he is 15 feet from the light, at what rate is the tip of his shadow moving? At what rate is the length of his shadow changing?

$$\frac{dx}{dt} = -5 \text{ ft/sec}$$

$$\frac{dy}{dt} - \frac{dx}{dt} = \frac{-125}{19} - (-5) = \frac{-30}{19} \text{ ft/sec}$$

Find  $\frac{dy}{dt}$  and  $\frac{d(y-x)}{dt}$  when  $x=15$  ft

$$\frac{25}{y} = \frac{6}{y-x}$$

$$-19y = -25x$$

$$\frac{dy}{dt} = \frac{25}{19} \frac{dx}{dt} = \frac{25}{19} (-5)$$

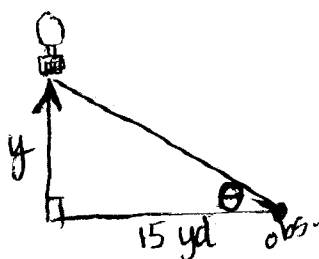
$$y = \frac{25}{19}x$$

$$\frac{dy}{dt} = -\frac{125}{19} \text{ ft/sec}$$

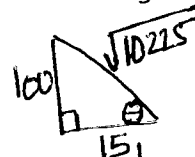
$$6y = 25y - 25x$$

### Example 5

A hot air balloon is rising at a rate of 9 yards per second. From a point 15 yards away on the ground, an observer measures the height of the balloon. What is the rate of change of the angle from the ground to the balloon when the balloon is 100 yards high?



$$\frac{dy}{dt} = 9 \text{ yd/sec}$$



Find  $\frac{d\theta}{dt}$  when  $y=100$  yds

$$\tan \theta = \frac{y}{15}$$

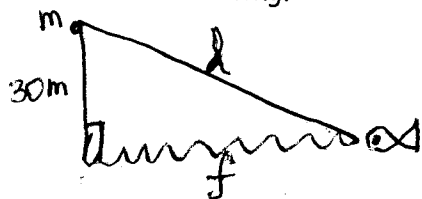
$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{15} \frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{\frac{1}{15} \frac{dy}{dt}}{\sec^2 \theta}$$

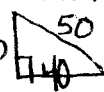
$$\frac{d\theta}{dt} = \frac{\frac{1}{15} (9)}{\left(\frac{\sqrt{10225}}{15}\right)^2} = \frac{27}{2045} = .013 \text{ rad/sec}$$

### Example 6

A fish is being reeled in at a rate of 2 m/sec (that is, a fishing line is being shortened by 2 m/sec) by the Old Man by the Sea. If he is sitting 30 meters above the water, how fast is the fish moving through the water when the line is 50 meters long?



$$\frac{dl}{dt} = -2 \text{ m/sec}$$



Find  $\frac{df}{dt}$  when  $l=50$  m

$$30^2 + f^2 = l^2$$

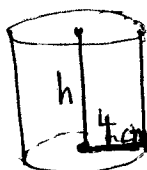
$$2l \frac{df}{dt} = 2l \frac{dl}{dt}$$

$$\frac{df}{dt} = \frac{2l \frac{dl}{dt}}{2f} = \frac{(50)(-2)}{40}$$

$$\frac{df}{dt} = -\frac{5}{2} \text{ m/sec}$$

### Example 7

The height of a cylinder with radius of 4 cm is increasing at a rate of 2 centimeters per minute. Find the rate of change of the volume of the cylinder with respect to the time when the height is 10 centimeters.



$$\frac{dh}{dt} = 2 \text{ cm/min}$$

Find  $\frac{dV}{dt}$  when  $h=10$  cm.

$$V = \pi r^2 h = \pi (4)^2 h = 16\pi h$$

$$\frac{dV}{dt} = 16\pi \frac{dh}{dt}$$

$$\frac{dV}{dt} = 16\pi (2) = 32\pi \text{ cm}^3/\text{min}$$