Pre-Calc Name: ______________________

Notes 9.4 (Part 1) – Introduction to Sequences

Definition: A sequence is an ordered progression of numbers. This progression can be finite (meaning it ends), for example{3, 6, 9, 12, …, 21}. Or, it can be infinite, for example {3, 6, 9, 12, ….}.

Notation: a_n is used to denote a term in a sequence. The a alone actually has no value, however the n has a very significant meaning. It indicates the place of the term in the sequence being referred to. a_5 5th term a_1 1st term a_0 initial term

There are 2 ways to define these sequences

explicitly & implicitly
(can find any term) (have to know the previous term to get the next term)

The **explicit definition** is like a formula.

Ex1) Find the first four terms of the given sequence.

a) $a_n = 2n+3$ arithmetic $d=2$

5, 7, 9, 11
a_1 a_2 a_3 a_4

b) $a_n = 3 \cdot 2^n$ geometric $r=2$

6, 12, 24, 48
a_1 a_2 a_3 a_4

c) $a_n = n + \frac{1}{n}$ neither

2, 2.5, $3\frac{1}{3}$, $4\frac{1}{4}$
a_1 a_2 a_3 a_4

NOW YOU TRY ☺ Find the first four terms of the given sequence.

a) $a_n = n^3 + 1$ neither

2, 9, 28, 65
a_1 a_2 a_3 a_4

b) $a_n = 3 - 7n$ arithmetic $d=-7$

-4, -11, -18, -25
a_1 a_2 a_3 a_4

c) $a_n = (-2)^n$ geometric $r=-2$

-2, 4, -8, 16
a_1 a_2 a_3 a_4

The **recursive definition** has 2 parts:

(1) a term to begin with
(2) a symbolic description of how the successive terms are related.

Ex2) Find the indicated terms of the given sequence.

a) $a_1 = 6$, $a_n = 4 + a_{n-1}$ previous term arith. $d=4$

10, 14, 18, 22
a_2 a_3 a_4 a_5

b) $a_1 = 9$, $a_n = \frac{1}{3} \cdot a_{n-1}$ geometric $r=\frac{1}{3}$

3, 1, $\frac{1}{3}$, $\frac{1}{9}$
a_2 a_3 a_4 a_5

c) $a_1 = 1, a_2 = 2$, $a_n = a_{n-1} + a_{n-2}$ neither

2, 3, 5, 8
a_2 a_3 a_4 a_5

NOW YOU TRY ☺ Find the indicated terms of the given sequence.

a) $a_1 = 4$, $a_n = 5 \cdot a_{n-1} + 2$

22, 112, 562, 2812
a_2 a_3 a_4 a_5

b) $a_1 = 1$, $a_n = \left(-\frac{1}{3}\right)^n \cdot a_{n-1}$

$\frac{1}{9}$, $-\frac{1}{243}$, $-\frac{1}{19683}$, $\frac{1}{4782969}$
a_2 a_3 a_4 a_5

$\left(-\frac{1}{3}\right)^2 \cdot 1 = \frac{1}{9}$ $\left(-\frac{1}{3}\right)^4 \cdot -\frac{1}{243}$

$\left(-\frac{1}{3}\right)^3 \cdot \frac{1}{9} = -\frac{1}{243}$ $\left(-\frac{1}{3}\right)^5 \cdot -\frac{1}{19683}$

c) $a_1 = 1, a_2 = 2$, $a_n = a_{n-1} \cdot a_{n-2}$

2, 2, 4, 8
a_2 a_3 a_4 a_5

Although it is possible to work with many different types of sequences, there are 2 that are most common.

arithmetic (where there is a common difference between each term) and geometric (where there is a common ratios between each pair of terms).

ARITHMETIC:

$a_n = a_1 + d(n-1)$, were d is the difference between each term (called the common difference)

Ex3) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the nth term of the sequence in terms of n.

a) 17, 21, 25, 29, …

type: arithmetic $d=4$

$a_n = 17+4(n-1)$

b) 8, 12, 18, 27, …

$r=1.5$

type: geometric

$a_n = 8(1.5)^{n-1}$

c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$ ($n=1$, $n=2$, $n=3$, $n=4$)

type: neither

$a_n = \frac{n}{n+1}$

d) 11, 101, 1001, 10001, …

type: neither

$a_n = 10^n+1$

NOW YOU TRY ☺ State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the nth term of the sequence in terms of n.

a) 100, –50, 25, –12.5,…

$r=\frac{-1}{2}$

type: geometric

$a_n = 100\left(-\frac{1}{2}\right)^{n-1}$

b) 1, 4, 9, 16, … ($n=1$, $n=2$, $n=3$, $n=4$)

type: neither

$a_n = n^2$

c) $\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$ ($n=1$, $n=2$, $n=3$, $n=4$)

type: neither

$a_n = \frac{n+1}{n^2}$

d) ~~2a – 2b, 3a – b, 4a, 5a + b, …~~

type: ______

$a_n =$ ______

Ex4) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the nth term of the sequence in terms of n.

a) $a_1 = 8,\ a_n = \frac{1}{2} \cdot a_{n-1}$

8, 4, 2, 1, …

type: geometric $r=.5$

$a_n = 8(.5)^{n-1}$

b) $a_1 = 6,\ a_n = a_{n-1} + 10$

6, 16, 26, 36, …

type: arithmetic $d=10$

$a_n = 6+10(n-1)$

c) $a_1 = \frac{1}{2},\ a_n = \frac{n}{n+1}(a_{n-1}+1)$

$\frac{2}{3}\cdot\frac{3}{2}$ $\quad \frac{3}{4}(2)=\frac{3}{2}$ $\quad \frac{4}{5}\left(\frac{5}{2}\right)$ $\quad \frac{5}{6}(3)$

$\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$

type: arithmetic $d=\frac{1}{2}$

$a_n = \frac{1}{2}+\frac{1}{2}(n-1)$

NOW YOU TRY ☺ State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the nth term of the sequence in terms of n.

a) $a_1 = 1,\ a_n = a_{n-1} + 2n - 1$

$1+3 \quad 4+5 \quad 9+7$

1, 4, 9, 16, …

type: neither

$a_n = n^2$

b) $a_1 = 3,\ a_n = -2 \cdot a_{n-1}$

3, –6, 12, –24, …

type: geometric $r=-2$

$a_n = 3(-2)^{n-1}$

c) $2^{\frac{2}{3}}, 2^{\frac{5}{3}}, 2^{\frac{8}{3}}, \ldots$

type: geometric $r=2$

$a_n = 2^{2/3}(2)^{n-1}$

$d=6$ $\quad d=-8$

Ex5) Find the indicated term of each arithmetic sequence: 7

a) $a_1 = 15,\ a_2 = 21,\ a_{20} = ?$

$a_n = a_1 + d(n-1)$

$a_{20} = 15 + 6(20-1)$

$\boxed{a_{20} = 129}$

b) $a_1 = 15,\ a_2 = \cancel{21}\ 7,\ a_{20} = ?$

$a_{20} = 15 + -8(20-1)$

$\boxed{a_{20} = -137}$

Ex6) How many terms are in the finite arithmetic sequence

a) 18, 24, …, 336 $\quad a_n = 336 \quad a_1 = 18 \quad d = 6$

$336 = 18 + 6(n-1)$

$\boxed{n = 54}$

b) 178, 170, …, 2

$2 = 178 + -8(n-1)$

$\boxed{n = 23}$

Ex7) Find the ~~number of multiples~~ r value of …

a) 7 between 30, and 300.

~~**b)** 6 between 28, and 280.~~

30, —, —, —, —, —, —, —, 300

1st term $\quad$ 9th term

$a_n = a_1(r)^{n-1}$

$300 = 30(r)^{9-1}$

$10 = r^8$

$r = \sqrt[8]{10}$

Ex 8) Find the

a) 100th term of the sequence
15, 12.3, 9.6, 6.9….

b) 120th ~~term of the sequence~~
-4, 2, 8, 14 ….

Ex9) Find the explicit definition for the sequences below:

a) $\frac{2}{5}, \frac{11}{15}, \frac{16}{15}, \frac{7}{5}, \ldots$

b) $\frac{7}{6}, \frac{5}{3}, \frac{13}{6}, \frac{8}{3}, \ldots$

c) -10, -6, -2, 2, . . .

d) -10.3, -6.5, -2.7, 1.1, . . .

Ex10)

a) Which term in the sequence 1, 4, 7,….is 88?

b) Which term in the sequence 1, 5, 9,….is 181?

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Notes (9.4 Part 2) – Geometric Sequences

Geometric Sequence: $a_n = a_1(r)^{n-1}$, where a_n is the nth term, r is the common ratio, & a_1 is the 1st term.

Ex1) Write an explicit representation of the pattern & state if it is arithmetic, geometric or neither. Then find the 15th term.

a) $\frac{1}{243}, \frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots$

geometric

$r = 3$

$a_{15} = \frac{1}{243}(3)^{15-1} = 19683$

$a_n = \frac{1}{243}(3)^{n-1}$

b) ~~53, 47, 41, 35, ...~~

$a_n =$ ______________

c) ~~2, 3, 5, 9, 17, 33, 65, ...~~

$a_n =$ ______________

Ex2) Given that $a_2 = 3$ & $a_5 = 24$ write an explicit formula if the sequence is a) arithmetic & b) geometric. Then find the values of a_3, and a_4 in each situation.

a) $-4, 3, 10, 17, 24$

$d = \frac{24-3}{3} = 7$

$a_n = -4 + 7(n-1)$

$a_3 =$ 10 $a_4 =$ 17

(These are called the Arithmetic means a_2 & a_5 between.)

b) $\frac{3}{2}, 3, 6, 12, 24$

$3r^3 = 24$

$r^3 = 8$

$r = 2$

$a_n = \frac{3}{2}(2)^{n-1}$

$a_3 =$ 6 $a_4 =$ 12

(These are called the Geometric means between a_2 & a_5)