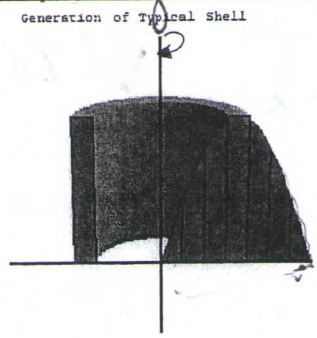
Notes ---- Volume: Shell Method

- The rectangle is parallel to the axis of revolution and perpendicular to the axis of integration.

- Formula is $V = 2\pi \int_a^b x f(x)\,dx$



Why is this method helpful?

$x^2(2-x)=0 \quad x=0, x=2$

- Suppose you have the region: $y = 2x^2 - x^3$ and $y = 0$. Revolve around the y-axis and find the volume of the solid. What is the issue?

can't solve for x

shell $V = 2\pi \int_0^2 x \cdot (2x^2 - x^3)\,dx = 2\pi \cdot \frac{8}{5}$

$= \boxed{\frac{16\pi}{5}}$

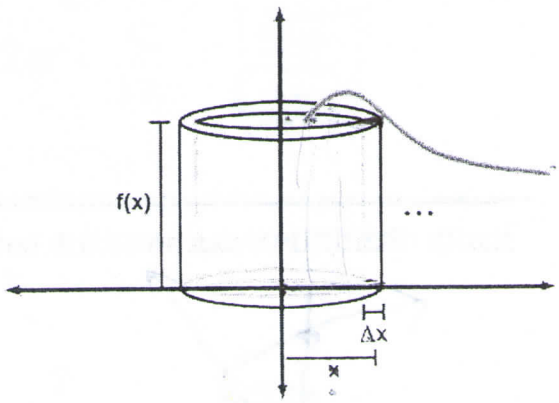
Deriving the Shell Method Formula

outer radius $= r_2$

inner radius $= r_1$

thickness of shell $= r_2 - r_1$

average radius $= \frac{r_1 + r_2}{2} = $ "r"



$V = \pi (r_2)^2 h - \pi (r_1)^2 h$

$= \pi h \left((r_2)^2 - (r_1)^2\right)$

$= \pi h (r_2 - r_1)\left(\frac{r_2 + r_1}{2}\right) \cdot 2$

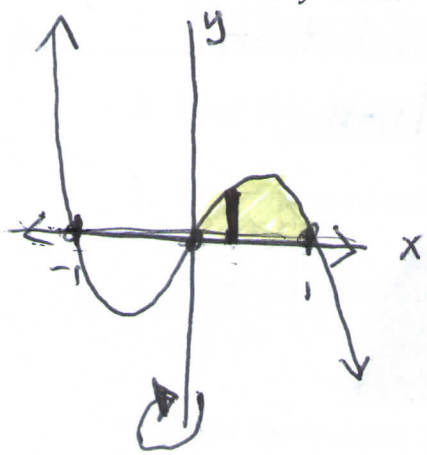
$= \pi h (r_2 - r_1)\, r \cdot 2$

$= 2\pi r h (r_2 - r_1)$

$2\pi \cdot$ radius $\cdot$ height $\cdot$ thickness

$V = 2\pi \int_a^b x \cdot f(x)\,dx$

Ex1) Find the volume of the solid: region is $y = x - x^3$, $y = 0$, from $x = 0$ to $x = 1$; revolve around the y-axis.



$x(1-x^2)=0$

$x(1+x)(1-x)=0$

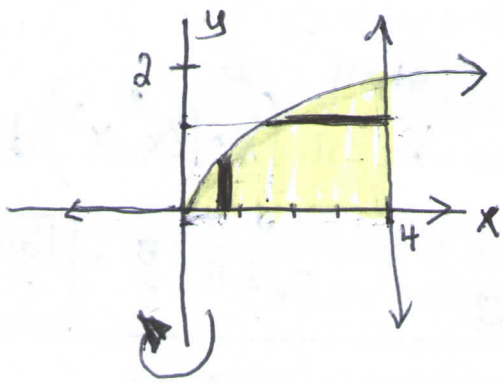
$x=0,-1,1$

Shell

$$V = 2\pi \int_0^1 x(x-x^3)\,dx = 2\pi \cdot \frac{2}{15} = \boxed{\frac{4\pi}{15}}$$

Ex 2) Find the volume of the solid: region is $y = \sqrt{x}$, $y = 0$, $x = 4$; revolve around the y-axis.

$\hookrightarrow x = y^2$



washer

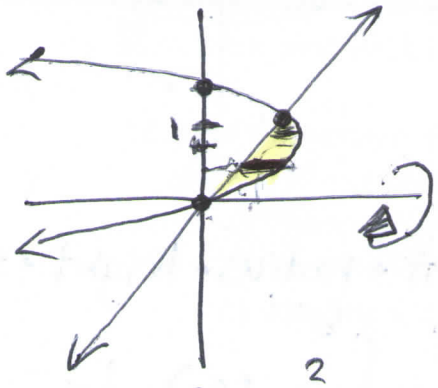
$$V = \pi \int_0^2 \left[(4)^2 - (y^2)^2\right] dy = \boxed{\frac{128}{5}\pi}$$

Shell

$$V = 2\pi \int_0^4 x\sqrt{x}\,dx = 2\pi \cdot 12.79999$$

$$= \boxed{80.425}$$

Ex 3) Find the volume of the solid: region is $x = 2y - y^2$, $x = y$; revolve around the x-axis.



$y(2-y)=0$

$y=0 \quad y=2$

Shell

$$V = 2\pi \int_0^1 y(2y-y^2-y)\,dy = 2\pi \cdot \frac{1}{12} = \boxed{\frac{\pi}{6}}$$

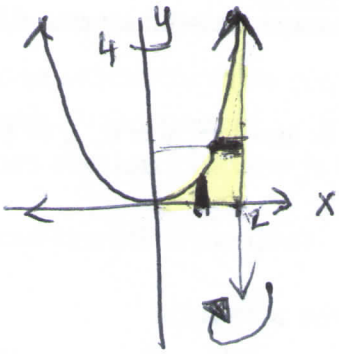
$2y-y^2 = y$

8 $\quad 0 = y^2 - y$

$0 = y(y-1) \quad y=0 \quad y=1$

Ex 4) Find the volume of the solid: region is $y = x^2$, $x = 2, y = 0$; revolve around $x = 2$.

$\hookrightarrow x = \pm\sqrt{y}$



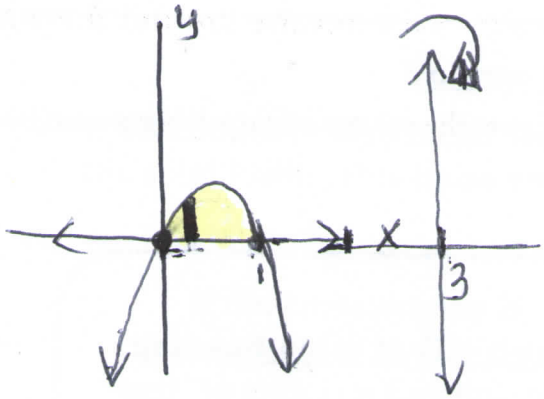
disk

$$V = \pi \int_0^4 (2-\sqrt{y})^2\, dy = \boxed{\frac{8\pi}{3}}$$

Shell

$$V = 2\pi \int_0^2 (2-x)x^2\, dx = 2\pi \cdot \frac{4}{3} = \boxed{\frac{8}{3}\pi}$$

Ex 5) Find the volume of the solid: region is $y = x - x^2$, $y = 0$; revolve around $x = 3$.



$x(1-x) = 0$

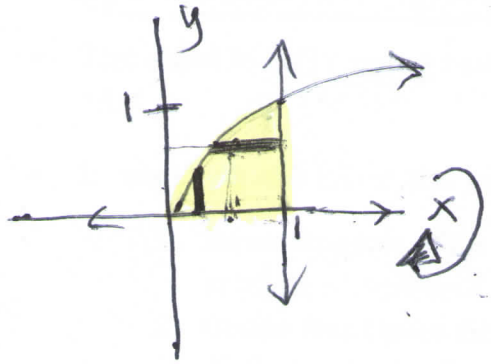
$x = 0 \quad x = 1$

shell

$$V = 2\pi \int_0^1 (3-x)(x-x^2)\, dx = 2\pi \cdot \frac{5}{12} = \boxed{\frac{5\pi}{6}}$$

Ex 6) Find the volume of the solid: region is $y = \sqrt{x}$, $y = 0$, $x = 1$; revolve around the x-axis

$\hookrightarrow x = y^2$



disk

$$V = \pi \int_0^1 (\sqrt{x})^2\, dx = \boxed{\frac{\pi}{2}}$$

shell

$$V = 2\pi \int_0^1 y(1-y^2)\, dy = 2\pi \cdot \frac{1}{4} = \boxed{\frac{\pi}{2}}$$