Notes Section 5.1 Part 1: Using Fundamental Trigonometric Identities

You are going to want to know by heart all of these identities! (Hint: Make flashcards!)

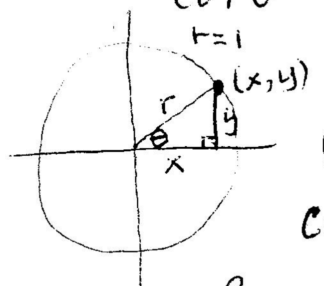
The Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad \cos\theta = \dfrac{1}{\sec\theta}$ $\qquad \tan\theta = \dfrac{1}{\cot\theta}$

The Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$



$x^2+y^2=r^2$

$(\cos\theta)^2+(\sin\theta)^2=(1)^2$

$\cos^2\theta+\sin^2\theta=1$

The Pythagorean Identities

$\cos^2\theta+\sin^2\theta=$ 1 $\qquad 1+\tan^2\theta=$ $\sec^2\theta$ $\qquad \cot^2\theta+1=$ $\csc^2\theta$

$\cos^2\theta = 1-\sin^2\theta$ $\qquad \cot^2\theta-\csc^2\theta=-1$

$\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$1+\tan^2\theta=\sec^2\theta$

The Co-function Identities: Accomplished by reflecting across the y-axis and a phase shift right $\frac{\pi}{2}$.

$\sin(\cancel{\theta}+\frac{\pi}{2}) = \sin(-1(\theta-\frac{\pi}{2}))$

$\dfrac{\cos^2\theta+\sin^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

$\cot^2\theta+1=\csc^2\theta$

$\sin\left(\dfrac{\pi}{2}-\theta\right)=$ $\cos\theta$ $\qquad \tan\left(\dfrac{\pi}{2}-\theta\right)=$ $\cot\theta$ $\qquad \sec\left(\dfrac{\pi}{2}-\theta\right)=$ $\csc\theta$

$\cos\left(\dfrac{\pi}{2}-\theta\right)=$ $\sin\theta$ $\qquad \cot\left(\dfrac{\pi}{2}-\theta\right)=$ $\tan\theta$ $\qquad \csc\left(\dfrac{\pi}{2}-\theta\right)=$ $\sec\theta$

The Odd/Even Identities: Recall symmetry proofs from Unit 1. Even functions are defined when $f(x)=f(-x)$ which shows symmetry about the y-axis. Odd functions are defined when $f(-x)=-f(x)$ which show symmetry about the origin.

$\sin(-\theta)=$ $-\sin\theta$ $\qquad \cos(-\theta)=$ $\cos\theta$ $\qquad \tan(-\theta)=$ $-\tan\theta$

$\csc(-\theta)=$ $-\csc\theta$ $\qquad \sec(-\theta)=$ $\sec\theta$ $\qquad \cot(-\theta)=$ $-\cot\theta$

All the identities above (and others) are used to rewrite expressions in terms of predetermined trig functions.

Examples of Rewriting Expressions:

1) Rewrite $\cot\theta\cos\theta$ in terms of $\sin\theta$

$\dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{\cos\theta}{1}$

$\dfrac{\cos^2\theta}{\sin\theta}$ $\qquad \dfrac{1-\sin^2\theta}{\sin\theta}$

2) Rewrite $\dfrac{\cot\theta}{\cos\theta}$ in terms of $\sin\theta$

$\dfrac{\;\frac{\cos\theta}{\sin\theta}\;}{\frac{\cos\theta}{1}} = \dfrac{\cancel{\cos\theta}}{\sin\theta}\cdot\dfrac{1}{\cancel{\cos\theta}} = \dfrac{1}{\sin\theta}$

plifying Expressions using Trigonometric substitutions and algebraic maneuvers:

When working out these identities, you can try one or more of the following techniques. I will explain each technique with examples:

1.) Change everything into sines and cosines.
2.) Use factoring to simplify the expression if possible.
3.) Get common denominators if there are fractions.
4.) Multiply both sides by a conjugate.

Of course, as we use the above techniques, be sure to refer back to the list of identities above. You might need to use some of them to simplify.

Use fundamental identities, arithmetic, and/or algebraic properties to simplify the following. Use no equal signs.

6) $\sin\theta\cot\theta$

$\frac{\cancel{\sin\theta}}{1}\cdot\frac{\cos\theta}{\cancel{\sin\theta}}$

$\boxed{\cos\theta}$

7) $cos^2\theta csc\theta sec\theta$

$\frac{\cos^{\cancel{2}}\theta}{1}\cdot\frac{1}{\sin\theta}\cdot\frac{1}{\cancel{\cos\theta}}$

$\frac{\cos\theta}{\sin\theta}$ $\boxed{\cot\theta}$

8) $sin\theta csc\theta cot\theta$

$\frac{\cancel{\sin\theta}}{1}\cdot\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cos\theta}{\sin\theta}$

$\frac{\cos\theta}{\sin\theta}$ $\boxed{\cot\theta}$

9) $cos\theta(1+tan^2\theta)$

$\cos\theta\cdot\sec^2\theta$

$\frac{\cancel{\cos\theta}}{1}\cdot\frac{1}{\cos^{\cancel{2}}\theta}$

$\frac{1}{\cos\theta}$ $\boxed{\sec\theta}$

10) $\frac{1-\cos^2\theta}{\cos^2\theta}$

$\frac{\sin^2\theta}{\cos^2\theta}$

$\boxed{\tan^2\theta}$

11) $\left(\csc^2\theta-1\right)\left(\sin^2\theta\right)$

$\cot^2\theta\cdot\sin^2\theta$

$\frac{\cos^2\theta}{\cancel{\sin^2\theta}}\cdot\cancel{\sin^2\theta}$

$\boxed{\cos^2\theta}$

12) $\sec(-x)\cos(-x)$

$\sec x\cdot\cos x$

$\frac{1}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{1}$

$\boxed{1}$

13) $\cot(-x)\cot\left(\frac{\pi}{2}-x\right)$

$-\cot x\cdot\tan x$

$-\frac{\cancel{\cos x}}{\cancel{\sin x}}\cdot\frac{\cancel{\sin x}}{\cancel{\cos x}}$

$\boxed{-1}$

14) $sin^3\theta+sin\theta cos^2\theta$

$\sin\theta\left(\sin^2\theta+\cos^2\theta\right)$

$\sin\theta\cdot 1$

$\boxed{\sin\theta}$

15) $\sin\theta-\tan\theta\cos\theta+\cos\left(\frac{\pi}{2}-\theta\right)$

$\sin\theta-\frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta}+\sin\theta$

$\cancel{\sin\theta}-\sin\theta+\sin\theta$

$\boxed{\sin\theta}$

16) $(1+sin\theta)(1-sin\theta)$

$1-\cancel{\sin\theta}+\cancel{\sin\theta}-\sin^2\theta$

$1-\sin^2\theta$

$\boxed{\cos^2\theta}$

17) $cot^2\theta-csc^2\theta$ $\boxed{-1}$

$\frac{\cos^2\theta}{\sin^2\theta}-\frac{1}{\sin^2\theta}$

$\frac{\cos^2\theta-1}{\sin^2\theta}$ $\frac{-\sin^2\theta}{\sin^2\theta}$ $\boxed{-1}$

18) $\frac{1}{cos^2x}-\frac{1}{cot^2x}$

$\sec^2x-\tan^2x$

$\boxed{1}$

19) $\frac{(\cos x)}{(\cos x)}\frac{(cosx)}{(1-sinx)}-\frac{(sinx)}{(cosx)}\frac{(1-\sin x)}{(1-\sin x)}$

LCD: $(1-\sin x)(\cos x)$

$\frac{\cos^2x-\sin x+\sin^2x}{\cos x(1-\sin x)}$

$\frac{\cancel{1-\sin x}}{\cos x\cancel{(1-\sin x)}}$

$\frac{1}{\cos x}$

$\boxed{\sec x}$

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