

# Integration by Substitution

11/13/18

review:  $\int x^{12} dx = \frac{1}{13} x^{13} + C$

$\int 2x(x^2+1)^{12} dx$  yucky!

- Steps:
1. Choose "u".
    - \* look for an "inside" function.
    - \* its derivative must be in the problem
  2. Calculate "du".
  3. Substitute in u & du.
    - \* all x's & dx must be replaced.
  4. Evaluate the new integral.
  5. Substitute to get "x" stuff again.

Evaluate:

①  $\int \underline{2x} (\underline{x^2+1})^{12} \underline{dx}$

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$du = 2x dx$

$\int u^{12} du = \frac{1}{13} u^{13} + C$

$\frac{1}{13} (x^2+1)^{13} + C$

$$\textcircled{2} \int t^3 \sqrt{t^4 + 5} dt$$

$$u = t^4 + 5$$

$$\frac{du}{dt} = 4t^3$$

$$du = 4t^3 dt$$

$$\frac{1}{4} du = t^3 dt$$

$$\frac{1}{4} \int \sqrt{u} du = \frac{1}{4} \int u^{\frac{1}{2}} du = \frac{1}{4} \cdot \frac{2}{3} u^{\frac{3}{2}} + C$$

$$\boxed{\frac{1}{6} (t^4 + 5)^{\frac{3}{2}} + C}$$

$$\textcircled{3} \int \frac{x}{(1-x^2)^4} dx$$

$$u = 1 - x^2$$

$$\frac{du}{dx} = -2x$$

$$du = -2x dx$$

$$-\frac{1}{2} du = x dx$$

$$-\frac{1}{2} \int \frac{1}{u^4} du = -\frac{1}{2} \int u^{-4} du$$

$$= \frac{1}{6} u^{-3} + C$$

$$= \frac{1}{6} (1-x^2)^{-3} + C$$

$$= \boxed{\frac{1}{6(1-x^2)^3} + C}$$

$$\textcircled{4} \int x \sin x^2 dx$$

$$u = x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx \quad \frac{1}{2} du = x dx$$

$$\frac{1}{2} \int \sin u du$$

$$\frac{1}{2} \cdot (-\cos u) + C$$

$$\boxed{-\frac{1}{2} \cos x^2 + C}$$

$$(5) \int \frac{(\tan x)^7}{\sec^2 x} dx$$

$$u = \tan x$$
$$\frac{du}{dx} = \sec^2 x$$
$$du = \sec^2 x dx$$

$$\int u^7 du = \frac{1}{8} u^8 + C$$
$$= \frac{1}{8} (\tan x)^8 + C$$
$$= \boxed{\frac{1}{8} \tan^8 x + C}$$

$$(6) \int \frac{2x+1}{\sqrt{x+4}} dx$$

$\rightarrow x = u - 4$

$$u = x + 4$$
$$\frac{du}{dx} = 1$$
$$du = dx$$

$$\int \frac{2(u-4)+1}{\sqrt{u}} du = \int \frac{2u-8+1}{\sqrt{u}} du$$
$$= \int \frac{2u-7}{\sqrt{u}} du = \int (2u^{\frac{1}{2}} - 7u^{-\frac{1}{2}}) du$$
$$= \frac{4}{3} u^{\frac{3}{2}} - 14u^{\frac{1}{2}} + C$$
$$= \boxed{\frac{4}{3} (x+4)^{\frac{3}{2}} - 14(x+4)^{\frac{1}{2}} + C}$$

$$(7) \int (x+7) \sqrt[3]{3-2x} dx$$

$$u = 3-2x \rightarrow \frac{u-3}{-2} = x$$

$$\frac{du}{dx} = -2$$

$$du = -2 dx$$

$$-\frac{1}{2} du = dx$$

$$-\frac{1}{2} \int \left( \frac{u-3}{-2} + 7 \right) \sqrt[3]{u} du$$

$$-\frac{1}{2} \int \left( -\frac{1}{2}u + \frac{3}{2} + 7 \right) \sqrt[3]{u} du$$

$$-\frac{1}{2} \int \left( -\frac{1}{2}u^{\frac{4}{3}} + \frac{17}{2}u^{\frac{1}{3}} \right) du$$

$$-\frac{1}{2} \left[ -\frac{3}{14}u^{\frac{7}{3}} + \frac{51}{8}u^{\frac{4}{3}} + C \right]$$

$$\frac{3}{28} (3-2x)^{\frac{7}{3}} - \frac{51}{16} (3-2x)^{\frac{4}{3}} + C$$

$$(8) \int_{-2}^4 x^2 (x^3+8)^2 dx$$

$$u = x^3+8$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int u^2 du = \frac{1}{3} \cdot \frac{1}{3} u^3 + C = \frac{1}{9} (x^3+8)^3 + C$$

$$= \frac{1}{9} (72)^3 + C - \left( \frac{1}{9} (0)^3 + C \right)$$

$$= \boxed{41472}$$

$$\frac{1}{3} \int_0^{72} u^2 du = \frac{1}{9} u^3 + C \Big|_0^{72} = \frac{1}{9} (72)^3 + C - \left( \frac{1}{9} (0)^3 + C \right)$$

$$\text{if } x=-2, u = (-2)^3+8=0$$

$$\text{if } x=4, u = (4)^3+8=72$$

$$= 41472$$