

A.3-----SIMPLIFYING RATIONAL EXPRESSIONS

- STEP 1:** factor completely in both num. & denom.
STEP 2: "divide out" FACTORS common to both num & denom.
STEP 3: state restrictions on the variable, including those that are no longer visible in the final answer

Simplify each of the following:

Ex1) $\frac{x^2+6x+5}{x^2+3x-10}$

$$\frac{(\cancel{x+5})(x+1)}{(\cancel{x+5})(x-2)}$$

$$\boxed{\frac{x+1}{x-2}} \quad x \neq 2, -5$$

Ex2) $\frac{6x^2-7x-3}{8x^2-2x-15}$

$$\frac{(3x+1)(\cancel{2x-3})}{(4x+5)(\cancel{2x-3})}$$

$$\boxed{\frac{3x+1}{4x+5}} \quad x \neq -\frac{5}{4}, \frac{3}{2}$$

Ex3) $\frac{5x}{25x^2+10x}$

$$\frac{\cancel{5x}}{\cancel{5x}(5x+2)}$$

$$\boxed{\frac{1}{5x+2}} \quad x \neq -\frac{2}{5}, 0$$

Ex4) $\frac{2x^2+5x-12}{15-10x}$

$$\frac{(\cancel{2x-3})(x+4)}{5(3-2x) \rightarrow -5(-3+2x)}$$

$$\boxed{\frac{x+4}{-5} \text{ or } -\frac{1}{5}(x+4)}$$

$$x \neq \frac{3}{2}$$

Ex5) $\frac{y^2-36}{2y^2-7y-30}$

$$\frac{(\cancel{y-6})(y+6)}{(2y+5)(\cancel{y-6})}$$

$$\boxed{\frac{y+6}{2y+5}} \quad y \neq -\frac{5}{2}, 6$$

*****Always remember to state restrictions!!!*****

We must state restrictions for anything that ever appears in the denominator, even if the factor cancels out and is not in the final step.

A.3-----Multiplying & Dividing RATIONAL EXPRESSIONS

The process of multiplying/dividing rational expressions is essentially identical to simplifying them. The main things you want to keep in mind are:

- #1 - only divide out **FACTORS** (things being multiplied)
- #2 - pay close attention to location (top/bottom)
- #3 - to \div by a fraction, * by the reciprocal

Multiplying Rational Expressions

Ex6) $\frac{4c^2 + 8cd + 4d^2}{8} \cdot \frac{3c - 3d}{c^2 - d^2}$

$4(c^2 + 2cd + d^2)$

$\frac{4(c+d)(c+d)}{8} \cdot \frac{3(c-d)}{(c+d)(c-d)}$

$\frac{12(c+d)}{2 \cdot 8} = \frac{3(c+d)}{2}$ $c+d \neq 0$ $c-d \neq 0$
 $c \neq -d$ $c \neq d$

Ex8) $\frac{x^2 - 9}{x^2 + 5x + 4} \div \frac{x^2 + 4x + 3}{x^2 + 7x + 12}$

$\frac{(x+3)(x-3)}{(x+4)(x+1)} \div \frac{(x+3)(x+1)}{(x+4)(x+3)}$

$\frac{(x+3)(x-3)}{(x+4)(x+1)} \cdot \frac{(x+4)(x+3)}{(x+3)(x+1)}$

$\frac{(x-3)(x+3)}{(x+1)^2}$ $x \neq -4, -1, -3$

Ex7) $\frac{x^2 - 6x - 7}{x^2 + 5x + 4} \times \frac{2x^2 + 9x + 4}{3x^2 - 23x + 14}$

$\frac{(x-7)(x+1)}{(x+4)(x+1)} \times \frac{(2x+1)(x+4)}{(3x-2)(x-7)}$

$\frac{2x+1}{3x-2}$ $x \neq -4, -1, \frac{2}{3}, 7$

Ex9) $\frac{3t^2 + 10t - 8}{4t^2 - 12t + 9} \div \frac{6t^2 - 13t + 6}{4t^2 - 9}$

$\frac{(3t-2)(t+4)}{(2t-3)(2t-3)} \cdot \frac{(2t+3)(2t-3)}{(3t-2)(2t-3)}$

$\frac{(t+4)(2t+3)}{(2t-3)^2}$ $x \neq \pm \frac{3}{2}, \frac{2}{3}$

You Try:

1) $\frac{x^2 - 3x}{x^2 - 9}$

2) $\frac{2x^2 + 11x - 21}{x^3 + 2x^2 + 4x} \cdot \frac{x^3 - 8}{x^2 + 5x - 14}$

3) $\frac{x^3 + 1}{x^2 - x - 2} \div \frac{x^2 - x + 1}{x^2 - 4x + 4}$

$\frac{x}{x+3}$

$x \neq \pm 3$

$\frac{2x-3}{x}$

$x \neq -7, 0, 2$

$x - 2$

$x \neq -1, 2$

Adding and Subtracting Rational Expressions

When we add or subtract fractions with common denominators, we add or subtract the numerators only.

$$\text{Ex10)} \frac{5}{2y} + \frac{7}{2y} = \frac{12}{2y} = \frac{6}{y}$$

In the situation that we do not start with a common denominator, we need to find one. We will need to look for the GCF of the two denominators essentially the same way that we do with simple fractions.

$$\text{Ex11)} \frac{3 \cdot 3}{3 \cdot 4} + \frac{1 \cdot 4}{3 \cdot 4} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12}$$

LCD = 12

STEP 1: Factor EVERYTHING YOU POSSIBLY CAN in BOTH the numerator and the denominator

STEP 2: Simplify at this point if possible (not very common)

STEP 3: Clearly state the Least Common Denominator (LCD)

STEP 4: Multiply (top & bottom) each rational expression as necessary to create common denominator

STEP 5: FOIL out the NUMERATOR ONLY, so that you can perform the addition/subtraction

STEP 6: Simplify (**Note: This might require more factoring and/or canceling)

STEP 7: State the restrictions

$$\text{Ex12)} \frac{2x-1}{2x^2+3x-2} + \frac{2x+5}{x^2+5x+6}$$

$$\frac{\cancel{2x-1} \cdot 1 \cdot (x+3)}{(\cancel{2x-1})(x+2)(x+3)} + \frac{2x+5}{(x+3)(x+2)}$$

$$\text{LCD: } (x+2)(x+3)$$

$$\frac{x+3 + 2x+5}{(x+2)(x+3)} = \frac{3x+8}{(x+2)(x+3)}$$

$x \neq \frac{1}{2}, -2, -3$

$$\text{Ex14)} \frac{x}{2x^2+7x-15} + \frac{x+1}{2x^2-11x+12}$$

$$\frac{x(x-4)}{(x-4)(2x-3)(x+5)} + \frac{(x+1)(x+5)}{(2x-3)(x-4)(x+5)}$$

$$\text{LCD: } (2x-3)(x+5)(x-4)$$

$$\frac{x^2-4x + x^2+1x+5x+5}{(x-4)(2x-3)(x+5)}$$

$$\frac{2x^2+2x+5}{(x-4)(2x-3)(x+5)} \quad x \neq \frac{3}{2}, -5, 4$$

$$\text{Ex13)} \frac{3}{3x+9} - \frac{16x}{4x-12}$$

$$\frac{3 \cdot 1(x-3)}{3(x+3)(x-3)} - \frac{4x \cdot (x+3)}{4(x-3)(x+3)}$$

$$\text{LCD: } (x+3)(x-3)$$

$$\frac{x-3 - 4x^2-12x}{(x+3)(x-3)} = \frac{-4x^2-11x-3}{(x+3)(x-3)}$$

$$\frac{-(4x^2+11x+3)}{(x+3)(x-3)} \quad x \neq \pm 3$$

$$\text{Ex15)} \frac{2}{x^2-2x} + \frac{1}{x} - \frac{3}{x^2-4}$$

$$\frac{2(x+2)}{x(x-2)(x+2)} + \frac{1(x+2)(x+2)}{x(x+2)(x+2)} - \frac{3(x)}{(x+2)(x-2)(x)}$$

$$\text{LCD: } x(x-2)(x+2)$$

$$\frac{2x+4 + x^2+2x-2x-4 - 3x}{x(x-2)(x+2)}$$

$$\frac{x^2-x}{x(x-2)(x+2)} = \frac{x(x-1)}{x(x-2)(x+2)}$$

$$\frac{x-1}{(x-2)(x+2)} \quad x \neq \pm 2, 0$$

Now You Try:

$$4) \frac{x+2}{4x} + \frac{x}{3x^2+9x}$$

$$\frac{3x^2+19x+18}{12x(x+3)}$$

$$x \neq 0, -3$$

$$5) \frac{2x}{x^2-1} - \frac{x+2}{x^2+2x+1}$$

$$\frac{x^2+x+2}{(x+1)^2(x-1)}$$

$$x \neq 1, -1$$

$$*6) \frac{3}{x} - \frac{5x}{x^3+1} + \frac{1}{x^2-1}$$

$$\frac{3x^4-7x^3+4x^2+4x-3}{x(x+1)(x-1)(x^2-x+1)}$$

$$x \neq \pm 1, 0$$

SIMPLIFYING COMPOUND FRACTIONS

The easiest way to work with compound fractions is to clearly identify a "top" and "bottom" and simplify what is on the "top" as if it were ITS OWN PROBLEM, meanwhile you will do the same thing with the "bottom." AFTER you have finished whatever must be done on the top & bottom, THEN you MULTIPLY BY THE RECIPROCAL

$$\text{Ex16) } \frac{3 - \frac{7}{x+2}}{1 - \frac{1}{x-3}}$$

$$\text{Ex17) } \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\frac{1}{a} - \frac{1}{b}}$$

Now You Try

$$7) \frac{\frac{3}{x+1}}{\frac{3}{x+1} - \frac{1}{x}}$$

$$8) \frac{\frac{x-2}{3x+1}}{\frac{1}{x} - \frac{2}{3x+1}}$$